MATRIX GEOMETRIC APPROACH FOR M/M/C/N QUEUE WITH TWO-PHASE SERVICE

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Abstract

An M/M/C/N Queue with Two-phase for exhaustive service without gating and server’s startups are studied in this paper. Customers arrive individually according to a Poisson process and receive batch service in the first phase and individual service as per exponential order in the second phase. After providing a second phase of service, the servers return to the batch queue to serve the customers who have arrived. Under the steady state conditions explicit expressions of the number of customers in the system are obtained and also derived the expected mean system length. The sensitivity analysis is also investigated graphically.

Index Terms: Two-Phase service, Steady-State and Mean System Length.

1. INTRODUCTION

Queues with two-phase system have been studied extensively from its own theoretical interest as well as its applicability to many engineering systems such as computers, communication networks, manufacturing systems and production systems etc.

There are many instances in which a server in a distributed system has to decide on resource allocation or handling some other problems where the decision is based on the state of the system. For example: Consider the situation of a production system, where the machine producing certain items may require two-phases of service in succession such as periodic checking (first phase of service) followed by a usual processing (second phase of service) to complete the processing of flaw materials. It may so happen that the machines may need to be stopped for overhauling after these two-phases of service.

The inventory problem in which the arriving orders are collected and when the order arrives, their service requirements, such as due date, quantity, and quality, are analyzed initially in batch mode. This is followed by individual services of the batch. As the system empties, which means there are no orders, stock can be replenished to prepare for the next orders.

This system can be utilized as model building of a scheduling problem, where all ships arriving at a port may need unloading service on arrival but only some of them may require reloading service soon after the unloading. There are many other situations in manufacturing process which involve two-phases of service.

The major emphasis on these models is laid on deriving the expressions for steady-state probabilities and the expected length of system. In this work an attempt is made to extend single server concept to multi server queue with two-phase service.

1.1 Queues with Two Phase Service

Many authors have studied two-phase of service in queuing systems. These queuing systems have got wide applications. Ostrovskie (1984) has studied A two-phase queuing system with an arbitrary number of priority Poisson input process is investigated. Krishna and Lee (1990), have studied some problems in distributed system control, such as load balancing, routing, scheduling in a real time environment, and reconfiguration require two-phase execution at a central server. Sivasamy et al. (2002) have studied a finite state multi server queuing system of the M/M/C type. Formulating the continuous time queue length process as Markov Process.
Krishna Kumar et al. (2002) had studied an M/G/1 retrial queuing system with additional phase of service and possible preemptive resume service discipline is considered. Kim et al. (2003) had studied the modeling and analyses of a discrete time, two-phase queuing system for both exhaustive batch service and gated batch service. Packets arrive at the system according to a Bernoulli process and receive batch service in the first-phase and individual services in the second phase.

Choudary (2003) examined the steady-state behavior of an M/G/1 queue with a second optional service in which the server may provide two-phases of heterogeneous service to incoming units. Artalejo and Choudhury (2004) had investigated the steady-state behavior of an M/G/1 queue with repeated attempts in which the server may provide an additional second phase of service. Choudhury and Madan (2004) considered a batch arrival queuing system, where the server provides two-phases of heterogeneous service one after the other to the arriving batches under Bernoulli schedule vacation. Choudhury and Paul (2005) have studied with an M/G/1 queue with two-phases of heterogeneous services and Bernoulli feedback system.

Madan and Choudhary (2005) have studied a single server queue with Poisson input, two-phases of heterogeneous service with Bernoulli schedule and a general vacation time. Gautam and Lotfi (2008) deals with the steady-state behavior of an M/G/1 queue with an additional second phase of optional service subject to breakdowns occurring randomly at any instant while serving the customers and delayed repair.

### 1.2 Matrix Geometric Method

Over the last two decades considerable amount of effort has been put in the development and the application of matrix geometric techniques for the analysis of queuing systems of which the (embedded) Markov chain exhibits a regular structure. A matrix geometric method first introduced by Neuts (1981) while studying the embedded Markov chains of many practical queuing systems.

Gaver et al. (1984) had studied an efficient computational approach to the analysis of finite birth and death models in a Markovian environment are given. Kuo-Hsiung and Ying-Chung (2002) have studied an M/M/R queuing system with finite capacity plus balking, reneging, and server breakdowns.

### 2. METHOD

In this Paper analyses Poisson arrivals and two-phase service system. In the first phase, the server gathers information about or otherwise computers (if the information is already available) the status of the system. In the second, it chooses an action based on that status information. We call the first phase the batch phase to emphasize that the server’s activity during that time relates to a selected set of the jobs in its batch queue. We call the second phase the individual phase.

For example, in load balancing, when jobs come into the allocating server (called the dispatcher), it probes the whole, or a part of, the distributed system to determine which processors are lightly loaded, and which are not. The incoming jobs are then routed to the lightly loaded processors in such a way as to even out the job loading. The batch service here is the probing action which generates information for the whole set of jobs in the dispatcher queue, and the individual service is the allocation of individual job processors. Some of the authors have studied two-phase service queuing systems. This type of queuing problems can be easily found in computer and telecommunication systems etc.

The objectives of this paper are:

i. To establish steady state equations to determine the steady state probability distribution of the number of units in the system.

ii. To derive expected number of units in the system.

iii. To perform sensitivity analysis through numerical experiments.

#### 2.1 The Model

The server is modeled as oscillating between two queues, a batch queue and an individual queue. Incoming jobs join the batch queue when the servers are doing batch service; it chooses a certain number of jobs from the batch queue does the batch service for all of them together and switches to the individual queue. Jobs whose batch phase has been completed can enter the individual queue. Jobs are assumed to arrive according to a Poisson process with rate λ. The individual service time per job is exponentially distributed with mean 1/μ. The batch service time is also assumed to be exponentially distributed with mean 1/β. Note that the batch service time is assumed to be independent of the batch size.

#### 2.2 Exhaustive Service without Gating

We assume that the system has a two-state Markov process, where the state (i, j) means the following:

1. If i=0, then the servers are doing batch processing and j is the number of jobs in the system (all of whom are in the batch queue).
2. If i > 0, then the servers are working on the individual service, and there are (i - 1) jobs in the batch queue and j jobs in the individual queue.

The differential difference equations are as follows:

\[
P_{0,0}(t+\Delta t) = P_{0,0}(t) (1-\lambda\Delta t) + P_{1,1}(t) \mu \Delta t \quad \ldots \ldots 1
\]

\[
P_{0,n}(t+\Delta t) = P_{0,n}(t) (1-\lambda) + P_{0,n+1}(t) \lambda \Delta t + P_{n+1,1}(t) (\mu + \lambda) \Delta t,
\text{for } n=1,2,3, \ldots (c-1). \quad \ldots \ldots 2
\]

\[
P_{0,n}(t+\Delta t) = P_{0,n}(t) (1-\lambda) + P_{0,n+1}(t) \lambda \Delta t + P_{n+1,1}(t) \mu \Delta t,
\text{for } n=c, c+1, \ldots \ldots (N-1). \quad \ldots \ldots 3
\]

\[
P_{0,N}(t+\Delta t) = P_{0,N}(t) (1-\mu) \Delta t + P_{N+1,1}(t) (\mu + \lambda) \Delta t. \quad \ldots \ldots 4
\]

\[
P_{1,1}(t+\Delta t) = P_{1,1}(t) (1-\lambda) + P_{0,0}(t) \mu \Delta t + P_{1,n+1}(t) (\mu + \lambda) \Delta t,
\text{for } n=1,2, \ldots \ldots (c-1). \quad \ldots \ldots 5
\]

\[
P_{1,n}(t+\Delta t) = P_{1,n}(t) (1-\lambda) + P_{0,n+1}(t) \lambda \Delta t + P_{n+1,1}(t) \mu \Delta t,
\text{for } n=c, c+1, \ldots \ldots (N-1). \quad \ldots \ldots 6
\]

\[
P_{1,N}(t+\Delta t) = P_{1,N}(t) (1-\lambda) + P_{0,N}(t) \mu \Delta t. \quad \ldots \ldots 7
\]

\[
P_{1,n}(t+\Delta t) = P_{1,n}(t) (1-\lambda) + P_{1,n+1}(t) (\mu + \lambda) \Delta t,
\text{for } n=1,2, \ldots \ldots (c-1), i=2,3, \ldots \ldots N. \quad \ldots \ldots 8
\]

\[
P_{1,n}(t+\Delta t) = P_{1,n}(t) (1-\lambda) + P_{1,n+1}(t) \lambda \Delta t + P_{n+1,1}(t) \mu \Delta t,
\text{for } n=c, c+1, \ldots \ldots (N-1), i=2,3, \ldots \ldots N. \quad \ldots \ldots 9
\]

\[
P_{1,n}(t+\Delta t) = P_{1,n}(t) (1-\lambda) + P_{1,n+1}(t) \lambda \Delta t,
\text{for } i=N+1, n=N. \quad \ldots \ldots 10
\]

\[
P_{1,n}(t+\Delta t) = P_{1,n}(t) (1-\lambda) + P_{1,n+1}(t) (\mu + \lambda) \Delta t,
\text{for } n=1,2, \ldots \ldots (c-1), i=N+1 \quad \ldots \ldots 11
\]

\[
P_{1,n}(t+\Delta t) = P_{1,n}(t) (1-\lambda) + P_{1,n+1}(t) \lambda \Delta t + P_{n+1,1}(t) \mu \Delta t,
\text{for } n=c, c+1, \ldots \ldots (N-1), i=N+1. \quad \ldots \ldots 12
\]

\[
P_{N+1,N}(t+\Delta t) = P_{N+1,N}(t) (1-\mu) \Delta t + P_{N,N}(t) \lambda \Delta t,
\text{for } n=N, i=N+1. \quad \ldots \ldots 13
\]

The steady state equations are as follows:

\[
\lambda P_{00} = \mu P_{11}. \quad \ldots \ldots 14
\]

\[
(\lambda + \mu) P_{0,n} = \lambda P_{0,n+1} + \mu P_{n+1,1}, \text{for } n=1,2, \ldots \ldots c-1 \quad \ldots \ldots 15
\]

\[
(\lambda + \mu) P_{0,n} = \lambda P_{0,n+1} + \mu P_{n+1,1}, \text{for } n=c, c+1, \ldots \ldots n-1 \quad \ldots \ldots 16
\]

\[
c\beta P_{o,N} = \lambda P_{o,N+1} + \mu P_{N+1,1}. \quad \ldots \ldots 17
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1}, \text{for } n=c, c+1, \ldots \ldots N-1 \quad \ldots \ldots 18
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1}, \text{for } n=c, c+1, \ldots \ldots N-1 \quad \ldots \ldots 19
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1}. \quad \ldots \ldots 20
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1} \quad \ldots \ldots 21
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1} \quad \ldots \ldots 22
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1} \quad \ldots \ldots 23
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1} \quad \ldots \ldots 24
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1} \quad \ldots \ldots 25
\]

\[
(\lambda + \mu) P_{i,n} = \lambda P_{i,n+1} + \mu P_{i,n+1} \quad \ldots \ldots 26
\]

Solutions of the above steady-state equations are analytically more complicated. Hence we can use matrix geometric method to find $P_{i,j}$ values.

### 2.3 Matrix Geometric Solution

A matrix geometric method was first introduced by Neuts (1981) while studying the embedded Markov chains of many practical queuing systems. Sivasamy et al. (2002) have studied a finite state multi-server queuing system. Using the matrix geometric method, we obtained the steady-state probabilities $P_{i,j}$; $1 \leq i \leq N + 1, 1 \leq j \leq C$. The corresponding steady-state matrix Q of this Markov chain has the block – tridiagonal form. The steady-state equations (14 to 26) can be written in the matrix Q form as shown in annexure.
2.4 Stationary Distribution

The stationary probability vector of Q matrix is given by

\[ \Pi_0 = [\Pi_0, \Pi_1, \Pi_2, \ldots, \Pi_{c-1}, \Pi_{c+1}, \ldots, \Pi_{c-1}]. \]
\[ \Pi_1 = [\Pi_0, \Pi_1, \Pi_2, \ldots, \Pi_{N-1}]. \]
\[ \Pi_2 = [\Pi_0]. \]

\[ e = (1, 1, 1, \ldots, 1)^\top \] is a column vector of ones of finite order of appropriate dimension. In order to determine the stationary probability distribution and movements of first passage times, denote the restriction of the original process Q. Observed during those intervals of time spent at level \( K_0 \) by \( S_1 \) before the original process Q enters level \( K_0 \) for the first time for \( n=0,1 \) and 2. So, \( S_0 \) and \( S_1 \) are clearly transient processes. \( S_2 \) is the restriction of the queue process to the states of \( K_2 \), which is an ergodic markov process. We denote by \( C_n \) the infinitesimal generator of the \( S_n \) process for \( n=0,1 \) and 2. Then \( C_n \) matrices are given by the following system of equations.

\[ C_0 = A_0. \]
\[ C_1 = A_1 + M_1(-C_0)^{-1} \Lambda_0. \]
\[ C_2 = A_2 + M_2(-C_1)^{-1} \Lambda_1. \]

**Annexure**

\[
Q = \begin{pmatrix}
-\lambda & \lambda & - & - & - & - & - & - & - & - \\
-\lambda+\beta & \lambda & - & - & - & - & - & - & - & - \\
-\lambda+2\beta & \lambda & - & - & - & - & - & - & - & - \\
-\lambda+3\beta & \lambda & - & - & - & - & - & - & - & - \\
-\lambda+s\beta & \lambda & - & - & - & - & - & - & - & - \\
\mu & - & - & - & - & - & - & - & - & - \\
\mu & - & - & - & - & - & - & - & - & - \\
\mu & - & - & - & - & - & - & - & - & - \\
\mu & - & - & - & - & - & - & - & - & - \\
\mu & - & - & - & - & - & - & - & - & - \\
\end{pmatrix}
\]

This matrix can be divided into three disjoint groups \( K_0, K_1, \) and \( K_2 \) such that \( K_0 = \{0, 1, 2, \ldots, C-1\} \), \( K_1 = \{C, C+1, C+2, \ldots, \ldots, N-1\} \) and \( K_2 = \{N\} \), so that the matrix \( Q = k_0 U k_1 U k_2 \). Then the matrix \( Q \) could be expressed in the tri diagonal form as given below.

As per Gaver et al. (1984) Algorithm is as follows:

1. Determine recursively the matrices \( C_n, n=0,1,2 \).
2. Solve the system \( \theta_2 C_2 = 0, \theta_2 e = 1 \) and compute \( \theta_2 \).
3. Compute recursively the vectors \( \theta_1 \) and \( \theta_0 \) in this order using

\[ \theta_1 = \theta_2 M_2(-C_1)^{-1} \text{ and } \theta_0 = \theta_1 M_1(-C_0)^{-1}. \]

4. Re-normalise the vector \( \theta = (\theta_0, \theta_1, \theta_2) \) so obtained and denote the result as

\[ \Pi = \{\Pi_0, \Pi_1, \Pi_2\}. \]

It is found that \( \Pi_0 \) is proportional to \( \theta_1 \) for \( j=0,1 \) and 2 so that

\[ \Pi_0 e + \Pi_1 e + \Pi_2 e = 1. \]

In all the above statements, the column vectors \( \theta_j = (\theta_0, \theta_1, \ldots, \theta_{c-2}, \theta_{c-1}) \) and \( \theta_1 = (\theta_0, \theta_1, \ldots, \theta_{N-2}, \theta_{N-1}) \) are vectors of sizes \( C \) and \( N-1 \) respectively, while \( \theta_2 = (\theta_N) \) has a single element. Finally, since \( \theta_0 \) and \( \Pi_0 \) are proportional to each other i.e. \( \Pi_0 = C \theta_0 \).
\[
\mathbf{Q} = \begin{pmatrix}
\mathbf{K0} & \mathbf{K1} & \mathbf{K2} \\
\mathbf{K0} & \mathbf{A0} & \mathbf{\Lambda_0} & 0 \\
\mathbf{K1} & \mathbf{M1} & \mathbf{A1} & \mathbf{\Lambda_1} \\
\mathbf{K2} & 0 & \mathbf{M2} & \mathbf{\Lambda_2}
\end{pmatrix}
\]

Where \( \mathbf{A_0} = \begin{pmatrix}
-\lambda & \lambda & - & - & - & - \\
- & - & \lambda & - & - & - \\
- & - & - & \lambda & - & - \\
- & - & - & - & \lambda & - \\
- & - & - & - & - & \lambda \\
- & - & - & - & - & - \\
- & - & - & - & - & - \\
- & - & - & - & - & - \\
- & - & - & - & - & - \\
- & - & - & - & - & - \\
- & - & - & - & - & - \\
- & - & - & - & - & - \\
\end{pmatrix}_{\text{C} \times \text{C}}
\]

\[
\begin{pmatrix}
\mathbf{C} & \mathbf{C}+1 & - & - & - & - & - & - & - & - & \text{N-2} & \text{N-1} \\
\mathbf{C} & - & - & \mathbf{C}+1 & - & - & - & - & - & - & - & - \\
\end{pmatrix}_{(\text{N-C-1}) \times (\text{N-C-1})}
\]

\[
\mathbf{M_1} = \begin{pmatrix}
\end{pmatrix}
\]

\[
\mathbf{A_2} = \begin{pmatrix}
\mathbf{-S\mu_1} & \mathbf{1x1} \\
\mathbf{M_2} = \begin{pmatrix}
0 \\
0 \\
\end{pmatrix} & \mathbf{-S\mu} & \mathbf{-S\mu_1} \\
\end{pmatrix}_{\text{1} \times (\text{N-C})}
\]
We obtain the following C as the proportionality constant from the normalizing conditions.

\[ C = \frac{1}{\theta_0 e + \theta_1 e + \theta_2 e} \]

Thus the stationary distribution \( \Pi = \{\Pi_0, \Pi_1, \Pi_2\} \) is obtained explicitly for the M/M/C/N queue with two-phase service system.

A computer program is developed (C Language) to solve \( P_{ij} \)'s.

### 2.5 Expected Mean System Length

The expected mean system length is given by

\[ \bar{N} = \sum_{i=0}^{N} (i + j)P_{i,j} + \sum_{j=0}^{N-1} P_{0,j} - 1. \]

### 2.6 Characteristics of the Measures of Effectiveness

In this Paper we study the effects of variation in i) \( \lambda \) ii) \( \mu \) iii) \( \beta \) iv) \( C \) v) \( N \).

i. **Effect of variation in ‘\( \lambda \)’**: In order to study the variation in \( \lambda \) on expected mean system length when \( \mu = 20, \beta = 15, C = 2, N = 30 \) and for different values of \( \lambda \), the expected mean system length are calculated, as they are presented in Table-1 and figure - 1. It is evident from Table-1 and figure - 1 that for fixed values of \( \mu, \beta, C \) and \( N \), the expected system length is increased.

![Graph](image-url)
ii. **Effect of variation in \( \mu \):** In order to study the variation in \( \mu \) on expected mean system length when \( \lambda = 10, \beta = 15, C = 2, N = 30 \) and for different values of \( \mu \), the expected mean system length are calculated, as they are presented in Table-2 and figure - 2. It is evident from Table-2 and figure - 2 that for fixed values of \( \lambda, \beta, C, N \), the expected mean system length is decreased.

**TABLE – 2:** Expected system length for different values of \( \mu \), when \( \lambda = 10, \beta = 15, C = 2, N = 30 \).

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.858335</td>
</tr>
<tr>
<td>16</td>
<td>1.855083</td>
</tr>
<tr>
<td>17</td>
<td>1.847702</td>
</tr>
<tr>
<td>18</td>
<td>1.837763</td>
</tr>
<tr>
<td>19</td>
<td>1.826314</td>
</tr>
<tr>
<td>20</td>
<td>1.814054</td>
</tr>
<tr>
<td>21</td>
<td>1.801449</td>
</tr>
<tr>
<td>22</td>
<td>1.78881</td>
</tr>
<tr>
<td>23</td>
<td>1.77634</td>
</tr>
<tr>
<td>24</td>
<td>1.764172</td>
</tr>
<tr>
<td>25</td>
<td>1.75239</td>
</tr>
</tbody>
</table>

**FIGURE – 2:** Expected system length for different values of \( \mu \), when \( \lambda = 10, \beta = 15, C = 2, N = 30 \).

iii. **Effect of variation in \( \beta \):** In order to study the variation in \( \beta \) on expected mean system length when \( \lambda = 10, \mu = 20, C = 2, N = 30 \) and for different values of \( \beta \), the expected mean system length are calculated, as they are presented in Table-3 and figure - 3. It is evident from Table-3 and figure - 3 that for fixed values of \( \lambda, \mu \), \( \beta, N \), the expected mean system length is decreased.

**TABLE – 3:** Expected system length for different values of \( \beta \), when \( \lambda = 10, \mu = 20, C = 2, N = 30 \).

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.80957</td>
</tr>
<tr>
<td>16</td>
<td>1.800359</td>
</tr>
<tr>
<td>17</td>
<td>1.795768</td>
</tr>
<tr>
<td>18</td>
<td>1.791245</td>
</tr>
<tr>
<td>19</td>
<td>1.786815</td>
</tr>
<tr>
<td>20</td>
<td>1.782494</td>
</tr>
<tr>
<td>21</td>
<td>1.778294</td>
</tr>
<tr>
<td>22</td>
<td>1.774221</td>
</tr>
<tr>
<td>23</td>
<td>1.770279</td>
</tr>
<tr>
<td>24</td>
<td>1.766314</td>
</tr>
<tr>
<td>25</td>
<td>1.76239</td>
</tr>
</tbody>
</table>

**FIGURE – 3:** Expected system length for different values of \( \beta \). When \( \lambda = 10, \mu = 20, C = 2, N = 30 \).

iv. **Effect of variation in \( C \):** In order to study the variation in \( C \) on expected mean system length when \( \lambda = 10, \mu = 20, \beta = 15, N = 30 \) and for different values of \( C \), the expected mean system length are calculated as they are presented in Table-4 and figure - 4. It is evident from Table-4 and figure - 4 that for fixed values of \( \lambda, \mu, \beta, N \), the expected mean system length is decreased to certain stage.

**TABLE – 4:** Expected system length for different values of \( C \), when \( \lambda = 10, \mu = 20, \beta = 15, N = 30 \).

<table>
<thead>
<tr>
<th>( C )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.814054</td>
</tr>
<tr>
<td>3</td>
<td>1.47623</td>
</tr>
<tr>
<td>4</td>
<td>1.17596</td>
</tr>
</tbody>
</table>

**FIGURE – 4:** Expected system length for different values of \( C \).
**FIGURE – 4:** Expected system length for different values of C, when \( \lambda=10, \mu=20, \beta=15, N=30. \)

<table>
<thead>
<tr>
<th>N</th>
<th>N’</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.815661</td>
</tr>
<tr>
<td>10</td>
<td>1.823886</td>
</tr>
<tr>
<td>15</td>
<td>1.829689</td>
</tr>
<tr>
<td>20</td>
<td>1.830062</td>
</tr>
<tr>
<td>25</td>
<td>1.834479</td>
</tr>
<tr>
<td>30</td>
<td>1.83448</td>
</tr>
<tr>
<td>35</td>
<td>1.834481</td>
</tr>
<tr>
<td>40</td>
<td>1.83448</td>
</tr>
</tbody>
</table>

**FIGURE – 5:** Expected system length for different values of \( N \), when \( \lambda=10, \mu=20, \beta=25, C=2. \)

### 3. CONCLUSION

In this Paper “An M/M/C/N Queue with Two-phase Service System” is studied. We extended single server concept to the multi-server queue with two-phase service. We obtained steady-state equations and calculated the expected mean system length. Finally some numerical results for expected system length are studied.

### REFERENCES


**BIOGRAPHIES**

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