SEMI ANALYTICAL METHOD FOR PREDICTION OF SATELLITE ORBIT

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Abstract

A semi analytical method was presented for prediction of the satellite orbit. The short and long variations were investigated, in terms of satellite position and velocity for non spherical Earth's gravitational field. This method was used to avoid singularities in periodic terms of elements variation. A code was constructed to obtain numerical results which were compared with other methods.

***Keywords**: orbital elements, Earth gravitational field, perturbation methods, and orbit prediction.*

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1. INTRODUCTION

Kozai (1959) has used analytical solutions for perturbations due to non-spherical Earth in terms of J_2, J_3, J_4 . The techniques depend on the computations of mean element at epoch time t_0, then the variation of elements which enable to obtain position and velocity at prediction time t. These techniques were adopted in the soft ware package SGP8, which was used in North American Air Defense Command (NORAD) for space surveillance (Liu Lin et al, 2006), who has obtained analytical method strictly separates the variations of δr and δv caused by the long periodic terms from those caused by short periodic terms. In this work the position r and the velocity v vectors were extrapolated at epoch time t_0, the variation of elements expanded up to $O(e^2)$, then the position and velocity vectors were computed at prediction time t, the orbital elements were obtained at this time.

2. METHOD OF VARIATION

The components of the radius vector $\bar{r}$ and the velocity vector $\bar{v}$ in the Cartesian coordinates (x_1, x_2, x_3) and $(\dot{x}_1, \dot{x}_2, \dot{x}_3)$ respectively are defined as, Deutsch (1963).

$$x_1 = P1\, a\,(\cos E - e) + P2\, a\sqrt{1-e^2}\sin E \tag{1.1}$$

$$x_2 = Q1\, a\,(\cos E - e) + Q2\, a\sqrt{1-e^2}\sin E \tag{1.2}$$

$$x_3 = R1\, a\,(\cos E - e) + R2\, a\sqrt{1-e^2}\sin E \tag{1.3}$$

And

$$\dot{x}_1 = \frac{a^2\, n}{r}\left(-P1\sin E + P2\sqrt{1-e^2}\cos E\right) \tag{2.1}$$

$$\dot{x}_2 = \frac{a^2\, n}{r}\left(-Q1\sin E + Q2\sqrt{1-e^2}\cos E\right) \tag{2.2}$$

$$\dot{x}_3 = \frac{a^2\, n}{r}\left(-R1\sin E + R2\sqrt{1-e^2}\cos E\right) \tag{2.3}$$

Where n, a, e, and E are the mean motion, the semi-major axis of the orbit, the eccentricity, and the eccentric anomaly respectively, and $(\bar{P}, \bar{Q}, \bar{R})$ are defined as Deutsch (1963).

$$P1 = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i \tag{3.1}$$

$$P2 = -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i \tag{3.2}$$

$$P3 = \sin\omega\sin\Omega\sin i \tag{3.3}$$

$$Q1 = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \tag{3.4}$$

$$Q2 = -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i \tag{3.5}$$

$$Q3 = -\cos\Omega\sin i \tag{3.6}$$

$$R1 = \sin\omega\sin i \tag{3.7}$$

$$R2 = \cos\omega\sin i \tag{3.8}$$

$$R1 = \cos i \tag{3.9}$$

Where Ω, ω, i are the longitude of ascending nodes, the argument of epicenter, and the inclination of the orbital plane respectively.

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3. VARIATION OF ELEMENTS

A direct approach to a theory of an artificial satellite motion is to use the variation of elements method. The potential function is defined as

$$U = \frac{\mu}{r}\left[1 - \sum_{k=2}^{\infty} \frac{J_k}{r^k} P_k(\sin\delta)\right] \tag{4}$$

Where μ is related to the gravitational constant, J_k are the zonal coefficient, $P_k(\sin\delta)$ are Legendre Polynomials of argument $\sin\delta$ and order k, and r is the radius of the orbit. The potential function (4) truncated at k = 4. The zero order solution corresponds to

$$U_0 = -\frac{\mu}{r}.$$

The disturbing function F can be written as

$$F = U - U_0 = F1 + F2 + F3 + F4 \tag{5}$$

Where

$$F1 = \mu \frac{3}{2}\frac{J_2}{a^3}\left(\frac{1}{2}(\sin i)^2 - \frac{1}{3}\right)(1 - e^2)^{\frac{-3}{2}} \tag{6.1}$$

$$F2 = -\mu \frac{85}{8}\frac{J_4}{a^5}\left(\frac{3}{35}\frac{3}{7}(\sin i)^2 + \frac{3}{8}sin^4\, i\right)\left(1 + \frac{3\,e^2}{2}\right)(1 - e2)- 72$$
(6.2)

$$F3 = \mu\left[\frac{3}{2}\frac{J_3}{a^4}\sin i\left(\frac{5\,(\sin\, i)^2}{4} - 1\right) e\sin\omega\,(1 - e^2)^{\frac{-5}{2}} - \frac{35\,J_4}{8\,a^5}(\sin i)^3\left(\frac{9}{28} - \frac{3\,(\sin\, i)^2}{8}\right) e^3\cos 2\omega\,(1 - e^2)^{\frac{-7}{2}}\right] \tag{6.3}$$

$$F4 = -\mu\frac{3\,J_2}{a^3}\left(\frac{a}{r}\right)^3\left\{\left(\frac{(\sin\, i)^2}{2} - \frac{1}{3}\right)\left[1 - \left(\frac{a}{r}\right)^3(1 - e^2)^{\frac{-3}{2}}\right] + \frac{(\sin\, i)^2}{2}\cos(2f + 2\omega)\right\} \tag{6.4}$$

F1 is the first order disturbing function; F2 is the second order secular part of the disturbing function; F3 is the long periodic portion, and F4 is the short periodic portion of the disturbing function. Kozai(1959) proved that there are no long periodic perturbations of the first order in the expressions for the semi-major axis a of an artificial Earth satellite. Moreover, there are no secular terms in a.The detailed calculation of the element perturbations are accomplished by selecting the desired portion of the disturbing function F_j and using this function in the Lagrange's Gaussian Equations which are

$$\frac{da}{dt} = \frac{2}{n\,a}\frac{\partial F}{\partial M} \tag{7.1}$$

$$\frac{de}{dt} = \frac{1-e^2}{n\,a^2\,e}\frac{\partial F}{\partial M} - \frac{(1-e^2)^{\frac{1}{2}}}{n\,a^2\,e}\frac{\partial F}{\partial\omega} \tag{7.2}$$

$$\frac{di}{dt} = \frac{\cos\, i}{n\,a^2\sin\, i\sqrt{1-e^2}}\frac{\partial F}{\partial\omega} - \frac{1}{n\,a^2\sin\, i\sqrt{1-e^2}}\frac{\partial F}{\partial\Omega} \tag{7.3}$$

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{n\,a^2\,e}\frac{\partial F}{\partial e} - \frac{\cos\, i}{n\,a^2\sin\, i\sqrt{1-e^2}}\frac{\partial F}{\partial i} \tag{7.4}$$

$$\frac{d\Omega}{dt} = \frac{1}{n\,a^2\sin\, i\sqrt{1-e^2}}\frac{\partial F}{\partial i} \tag{7.5}$$

$$\frac{dM}{dt} = \frac{1-e^2}{n\,a^2\,e}\frac{\partial F}{\partial e} - \frac{2}{n\,a}\frac{\partial F}{\partial a} \tag{7.6}$$

Now, it is convenient to present a new method strictly separates the long periodic terms from the short periodic terms variations. This new method depends on the mean element $\bar{q}(t)$ at the prediction time t, which can be summarized as follows:

1- The mean elements $\bar{q}(t)$ are extrapolated at predicted time t from the mean elements $\overline{q_0}$ at epoch t_0.

2- $r(t)$, and $v(t)$ are converted to $r(\bar{q})$ and $v(\bar{q})$ respectively.

3- The periodic variations δr and δv at time t are computed.

4- The predicted orbital elements could be determined at the time $t + \delta t$.

A cod was constructed (Mathematicasoftware) to obtain the short periodic and the long periodic variations in r and v due to the effects of the zonal harmonics (J_2, J_3, J_4)of Earth's non – spherical gravitational potential on an artificial satellite of low altitude. This was done by using Equations (3) to (7) to obtain the expressions ($\delta a, \delta e, \delta i, \delta\Omega, \delta\omega, and\ \delta M$), terms of order O($e^2$) were retained, and the short periodic terms were strictly separated from the long periodic terms. Omitting the derivation, the expressions of the short periodic ($\delta x1_s, \delta x2_s, \delta x3_s, \delta\dot{x}1_s, \delta\dot{x}2_s, \delta\dot{x}3_s$) and long periodic variations($\delta x1_l, \delta x2_l, \delta x3_l, \delta\dot{x}1_l, \delta\dot{x}2_l, \delta\dot{x}3_l$) , in position and velocity respectively were obtained. The elements appearing in the obtained expressions are all mean elements. Finally, the expressions of variations $\delta r\ and\ \delta\dot{r}$ in position and velocity respectively could be obtained from

$$\delta r = \begin{pmatrix} \delta x1_s + \delta x1_l \\ \delta x2_s + \delta x2_l \\ \delta x3_s + \delta x3_l \end{pmatrix} \tag{8.1}$$

$$\delta\dot{r} = \begin{pmatrix} \delta\dot{x}1_s + \delta\dot{x}1_l \\ \delta\dot{x}2_s + \delta\dot{x}2_l \\ \delta\dot{x}3_s + \delta\dot{x}3_l \end{pmatrix} \tag{8.2}$$

4. RESULTS AND DISCUSION

The above perturbation solutions due to the dominant terms of the Earth's non-spherical gravitational potential (J_2, J_3, J_4) were extrapolated to a small eccentricity satellite orbit up to order $O(e^2)$, the prediction of the position r and velocity v were obtained at time t, and the prediction of orbital elements at this time were computed. For convenience of comparison, the results obtained by Lin et al (2006) were compared with the results obtained throughout this work. Table I illustrates the orbital elements of a low artificial satellite at time t_0, which were used to obtain the position r and velocity v at epoch t_0, while table II illustrates the comparison between the published results of Lin (2006) and the results obtained through this work.

Semi major axis (a) km	6942.7488
Eccentricity (e)	0.00634566
Inclination (i) degrees	98.2237
Longitude of ascending node (Ω) degrees	101.0486
Argument of perigee (ω) degrees	253.1687
Mean anomaly (M) degrees	106.8618

Table 1: Orbital elements of low artificial satellite.

Time and orbital element	Lin results (2006)	Results of this work
1-1440 m (a) km 2- 2880 m (a) km	6951.85992 6951.59982	6951.44893 6951.38745
1-1440 m (e) 2- 2880 m (e)	0.005259 0.005356	0.00618693 0.00592645
1-1440 m (i) deg. 2- 2880 m (i) deg.	98.218904 98.218258	98.2200756 98.2195328
1-1440 m (Ω) deg. 2- 2880 m (Ω) deg.	102.105520 103.162292	101.978542 102.908375
1-1440 m (ω) deg. 2- 2880 m (ω) deg.	250.850502 246.762971	251.268495 248.794382
1-1440m(ω+M) deg 2-2880m(ω+M) deg	355.911635 351.792314	M= 104.5192033 M= 101.8733514

Table 2: Comparison between Lin results (2006) And results of this work

Comparison of the results shows that our results in agreement of that obtained by Lin(2006), so it is convenient in the near future to apply the semi analytical method mentioned through this work on the prediction of the orbits of low Earth satellites taken into account the drag force, the gravitational force, and the indirect radiation pressure.

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