

FRACTAL APPLICATIONS IN ELECTRICAL AND ELECTRONICS ENGINEERING

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Abstract

Fractal, which means broken or irregular fragments, was originally coined by Dr B. Mandelbrot in the year the 1970. Fractals describe a family of complex shapes that possess an inherent self-similarity in their geometrical structure usually in nature, since the pioneering work of Mandelbrot and others, a wide variety of applications for fractals has been found in many branches of science and engineering. A fractal is a recursively generated object having a fractional dimension. Fractals are defined as the “set whose Hausdorff dimension exceeds topological dimension”. Fractal has various properties like recursive, infinite, self-symmetry and fractional dimension (FD). Self Symmetry, Space filling and FD are most important properties which has practical applications. This paper discuss various applications of fractal in electrical and electronics engineering field few are fractal antenna, fractal capacitor, fractal image compression, fractal encoding, fractal analysis in power systems fault analysis like (High Impedance fault, Load forecast). Hurst parameter is used for real dynamic curve analysis. Lightning modeling is essential for design and testing of an aircraft in laboratory. Full scale aircraft testing of lightning stroke inside lab fractal lightning modeling is used to generate High Volt lighting discharge testing of full model aircraft on ground. Impedance matching by impedance transformer by fractals is new concept for matching loads in electronics. L-System is extensively used for biological modeling and growth modeling. L-System is also used for artificial life creation by computers. L-System fractals can be used for digital camouflaged, concealed super structures, computer generated tree and artificial life. Scale free networks are fractal networks, they are more robust. Hurst parameter (H), of traffic for scale free networks if change abruptly, it signifies attack on network. It is also proposed to monitor H for scale free network. Routing protocol and optimization techniques in networks should also choose nodes so that the network grows and forms like fractal network which are robust.

Index Terms: FD, Correlation dimension, Self-symmetry, Space-filling, Hurst Parameter (H), L-System, fractal Radio, etc

1. INTRODUCTION

Our world and its real world problems are highly nonlinear and fractal, which cannot be described in terms of Euclidian geometry. Fractal geometry is considered fashionable mathematics, humanistic and quasi pragmatic. Besides, its connection with nature, art, science, engineering, the marvels of computer and IT development. Fractals geometry and modeling is present in almost all the natural creation. It has also impact on a special nature beauty, and a particular way of thinking, that can be felt in the mind and heart.

Fractal has important properties, like FD, self symmetry and space filling, which can be used for interpretation of real world nonlinear problems and systems. The dimension of the geometry can be interpreted as a quantification of the space filling ability of the geometry. While Euclidean geometries have integer dimensions, for example dimension of line is “1”, and “2” for a plane, but fractals have fractional dimensions.

Space filling curves after infinite iteration may achieve integer dimensions. Combining fractal geometry with dynamical systems lead to non-linear dynamical systems, which can help in solving problems in our current technological era that Einstein’s theory, was unable to solve problem like power system dynamics. Fig-1 shows iterations of Sierpinski fractals and Fig-2, various types of Fractals.

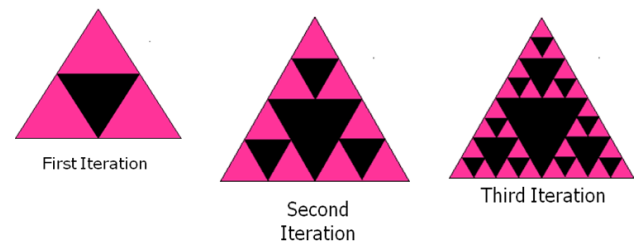


Fig-1: Iteration of Sierpinski Triangle

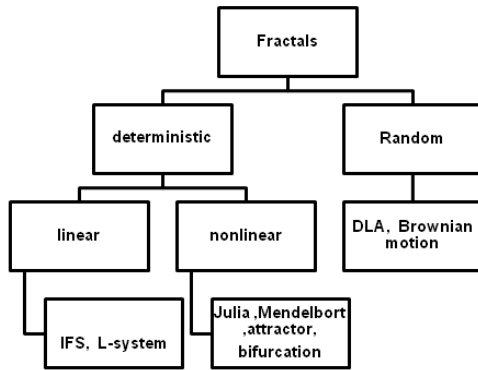


Fig-2: Types of Fractals

2. MATHEMATICAL BACKGROUND

FD can be defined in terms of various methods depending upon type of system become fractal. Box dimension method is usually for deterministic systems like measuring of coast line, Correlation dimension is for random system like measurement of dimension of strange attractor of dynamical system and for time series analysis “H” which is related to FD.

- Hausdorff dimension, how does the number of balls it takes to cover the fractal scale with the size and radius of the ball.
- Box-counting dimension, how does the number of boxes it takes to cover the fractal scale with the size of the boxes.
- Information dimension, how does the average information needed to identify an occupied box scale.
- Correlation dimension, calculated from the number of points used to generate the picture, and the number of pairs of points within a distance ε of each other.

If ‘s’ is the scale factor and Hausdorff dimension of an object is based on covering the object by small disks or balls for a minimum cover. The box dimension of a subset X of the plane is defined similarly by counting the number of cells of a grid with constant, ‘s’ that intersect X, N(s). Then ‘X’ has a dimension ‘D’ if, N(s) satisfies the power law.

$$N(s) = c(1/s)^D \tag{1}$$

Asymptotically in the sense that

$$\lim_{s \rightarrow 0} N(s) s^D = c. \tag{2}$$

The box dimension D can be computed from (2) as

$$D = \lim_{s \rightarrow 0} [- \log N(s)/\log(s)]. \tag{3}$$

Fractal dimension as defined by Equation (3) is usually identified with Hausdorff–Besicovitch dimension and is known as the capacity dimension. There is a fine distinction between the Hausdorff–Besicovitch dimension and the capacity dimension while the former is obtained by covering the set minimally with hyper cubes that may be different in size; the latter is obtained with the same process except that the hyper cubes are the same size. Spatial correlation is measured in terms of correlation dimension.

$$d_C = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log r},$$

where the correlation integral, C(r), is given by

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{j=1}^N \sum_{i=j+1}^N H(r - \|R_i - R_j\|),$$

“H” is Heaviside function.

Hurst parameter (H) is related with roughness of dynamic time series curve like Load curve of power plant, and can be used as a measure of FD. Estimating the Hurst exponent for a data set provides a measure of whether the data is a pure random walk or has underlying trends All the known representations of such bursts in processes have one common parameter value. This parameter has different but equivalent representations and names. That is H, FD, and the exponent “γ” of the 1/f process power spectrum are related as

$$\gamma = 2H - 1 = 5 - 2D$$

There is also a form of self-similarity called statistical self-similarity. Assuming that we had one of those imaginary infinite self-similar data sets, any section of the data set would have the same statistical properties as any other. Statistical self-similarity occurs in a surprising number of areas in engineering, computer network traffic traces are self-similar. Other examples of statistical self-similarity exist in cartography (the measurement of coast lines), computer graphics (the simulation of mountains and hills), biology (measurement of the boundary of a mould colony) and medicine (measurement of neuronal growth). Self-symmetry means superstructure is formed from small structure which is having scale invariance.

3. APPLICATIONS IN ELECTRICAL AND ELECTRONICS ENGINEERING

There are various applications of fractals in electrical and electronics engineering field some are:

3.1 High Impedance Fault Detection

In Power Systems, high impedance faults are very peculiar, these faults don't draw enough current to operate protective relays thus very difficult to predict. Phase currents and voltages in a distribution power system change with a certain degree of chaos when high impedance faults (HIFs) occur concepts of fractal geometry to analyze chaotic properties of high impedance faults. These faults also fail to establish return path, as fault progress it melts conductor, causes arcing, displacement of soil etc causes current very chaotic. FD is measure of this fault as shown in Fig3.

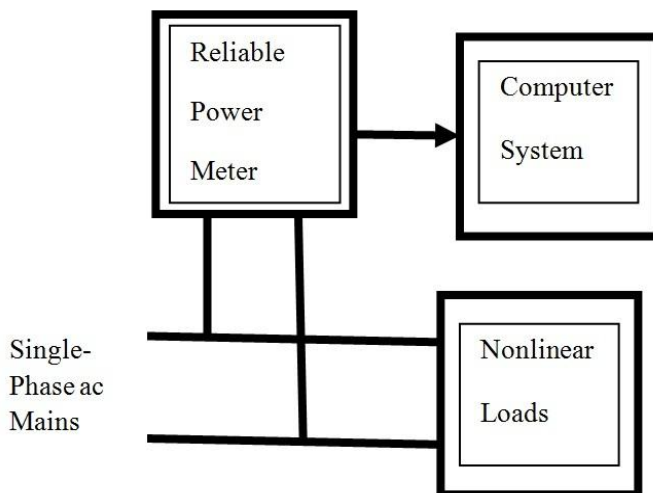


Fig-3: Block Diagram of Fault Locator

3.2 Fractal Capacitor

Unlike conventional metal-to metal capacitors, the density of a fractal capacitor increases with scaling. The capacitance per unit area of a fractal structure depends on the dimension of the fractal. To improve the density of the layout, fractals with large dimensions should be used. Fractals add one more degree of freedom to the design of capacitors, allowing the capacitance density to be traded for a lower series resistance. Most of the fractal geometries randomize the direction of the current flow and thus reduce the effective series inductance; fractals usually have lots of rough edges that accumulate electrostatic energy more efficiently compared to inter digitized capacitors.

3.3 Lightning Modelling

Lightning modelling is realised with fractal. Lightning modelling is used various applications. Study of lightning stroke on full scale model of aircraft on ground requires lightning stroke which is generated by fractal modelling. High voltage equipment full scale testing, and dielectric breakdown studies uses FD as a parameter.

3.4 Load Scheduling

Load curve is highly dynamic and depending on of various random parameters like, temperature, humidity, time of the day and social parameters like festivals etc. Roughness of curve can be linked with FD. Roughness as measure can be used by Artificial Intelligence (AI) systems for load scheduling.

3.5 Fractal Antenna Multiband/ Miniaturization

The next major opportunity for wireless component manufacturers is to put the antenna into the package [1]. Fractal-technology applied to antennas reaches the required miniaturization to make Full Wireless System in Package (FWSiP) a reality [2]. The space filling properties of fractals enable the production of miniature antennas with optimal performance, and with multi band capabilities. Fractal Antenna technology is suitable for Bluetooth, WLAN, GPS, UWB and Zigbee and for sensors for automotive, biomedical and industrial purposes. Some key benefits of fractals in antenna geometry are:

- Broadband and multiband frequency response that derives from the inherent properties of the fractal geometry of the antenna.
- Compact size compared to antennas of conventional designs (miniaturization), while maintaining good to excellent efficiencies and gains.
- Mechanical simplicity (no matching) and robustness. Characteristics of the fractal antenna are obtained due to its geometry and not by the addition of discrete components.
- Design to particular multi-frequency characteristics containing specified stop bands as well as specific multiple pass bands.

Apollonian gasket fractal antenna with CPW (Co-planer waveguide) monopole feed shown in Fig-4. The antenna is designed and fabricated on FR4 material with dielectric constant (4.33). Experimental result in laboratory condition

measured return loss (S11 -10 dB parameter) by Vector analyzer shows the multiband behavior with the centre frequencies of 1.265 GHz, 4.66 GHz and 7.8 GHz with bandwidth of 50%, 17.5% and 15 % respectively. This multiband is achieved with fractal geometry [3].

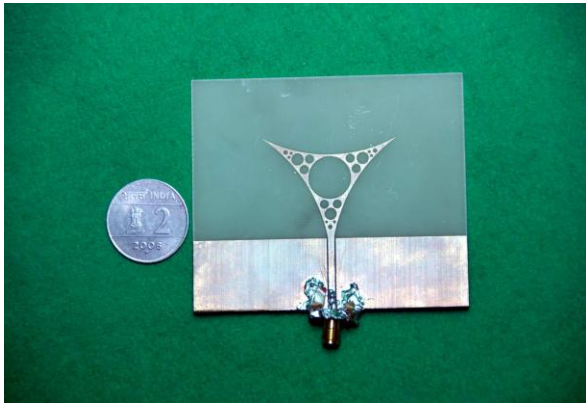


Fig-4: Apollonian Gasket CPW monopole Fractal Antenna

3.7 Fractal Image Compression

The algorithm involved in compressing an image is

- Specifying the rate (bits available) and distortion (tolerable error) parameters for the target image.
- Dividing the image data into various classes, based on their importance.
- Dividing the available bit budget among these classes, such that the distortion is a minimum.
- Quantize each class separately using the bit allocation information derived in step 3.
- Encode each class separately using an entropy coder and write to the file.

3.8 Impedance Transformer

Using a Sierpinski fractal micro-strip stepped-impedance transformers using a Sierpinski fractal shape are proposed for the first time as shown in Fig-6. The proposed fractal- shaped stepped-impedance transformers can greatly enhance the operating frequency bandwidth. Fractal Shape has enhanced bandwidth up to 44.0% and 73.9% (S11<30 dB) has been achieved by the presented two-section and three-section fractal-shaped maximal flat stepped-impedance transformers [5]. Z_1, Z_2, Z_3 are Impedances, W_1, W_2, W_3 are Width and $\lambda g_1, \lambda g_2, \lambda g_3$ are wavelengths where, $l_1 = \frac{\lambda g_1}{4}, l_2 = \frac{\lambda g_2}{4}, l_3 = \frac{\lambda g_3}{4}$.

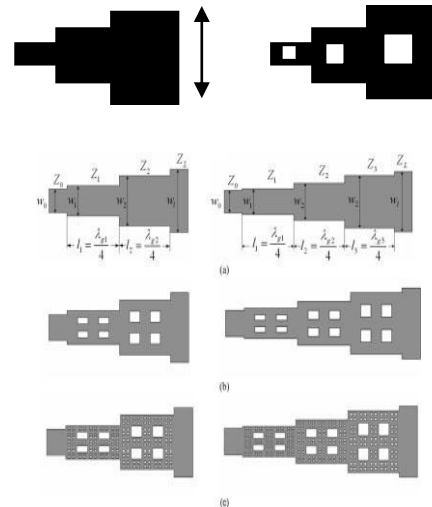


Fig-6 Fractal Impedance Transformer for matching load

3.6 Fractal Encoding in Communication

Fractal-coded images are decompressed with a low-complexity decoding scheme [4]. The fractal-based compression utilizes local self similarity in images of textures and natural scenes. Fractal image textures offer significant savings in both storage and transmission bandwidth. Fractal compression/decompression could surpass the JPEG standard in mobile applications as shown in Fig-5.

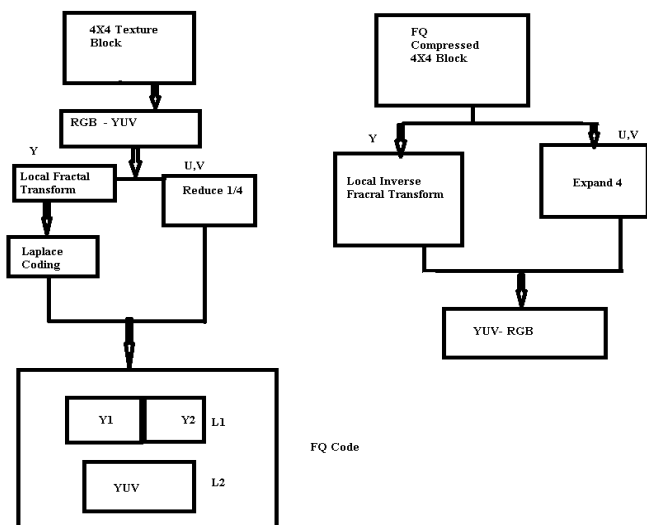


Fig-5: Fractal Coding

3.9 Artificial Life (Computer Graphics)

Fractals are commonly used in computer graphics, not only for creating visual effects, such as fires, clouds, and lightning, but also for modeling living organisms. Lindenmayer system (L-system) is a formal grammar (a set of rules and symbols) most famously used to model the growth processes of artificial plant development as shown in Fig-7, it is used to model the morphology of a variety of organisms [6]. Furthermore, the interest of biologists in modeling actual plant species is complemented by the fundamental studies of emergence in the field of artificial life. These varied interests and applications place L-systems in the center of interdisciplinary studies bridging theoretical computer science, computer graphics, biology and artificial life. Commands for Turtle action are as shown in Table-1.

Symbol	Meaning
F	Draw forward by a fixed length.
G	Go forward without drawing a line.
+	Turn right by a fixed angle, n+ means turn n times.
-	Turn left by a fixed angle, n- means turn n times.
[Push (put) current position to stack.
]	Pop (take) current position from stack.
	Draw forward by a length that depends on the execution depth.

Table-1: Commands for Turtle Grammar

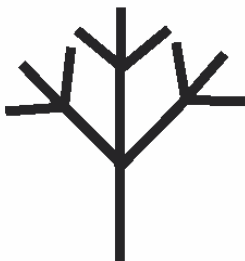


Fig-7 Fractal L-System (Artificial Plant)

Fibonacci sequence is used by most of the biological modeling by L-System as shown in Table-2.

Initial state: A: B

B: AB

And so on....1 1 2 3 5 8 13

n	Series
n=0	A
n=1	AB
n=3	BAB
n=4	ABBAB
n=5	BABABBAB

Table-2: Fibonacci Series

3.10 Fractal Radio and Fractal Radar Concept

Application of fractal theory in combination with the theory of fractional integrodifferential operators and fractal interpretation to various problems of modern radio engineering is widely accepted. Fractal methods used to process very weak signals and low-contrast images on the basis of application of non-Gaussian statistics, to measure new attributes of signals scattered by a statistically rough surface, and to consider scaling effects of real radio signals and electromagnetic fields can be used for the development of new efficient fractal electronic systems with new capabilities, which are sometimes unreachable for the existing facilities designed on the basis of principles of traditional electronics.

In Fractal Radar, tone and texture and structure features on the radar imagery are important bases of target recognition, FD is used to extract the texture and structural features of ground object.

3.11 Scale Free Networks

Social behaviour and spread of internet as an outcome of social behaviour and communications leads to scale free networks called Francnet [7]. A scale-free network is a network whose degree distribution follows a power law, at least asymptotically. That is, the fraction $P(k)$ of nodes in the network having k connections to other nodes goes for large values of k as

$$P(k) \sim ck^{-\gamma}$$

Where " c " is a normalization constant and " γ " is a parameter whose value is typically in the range $2 < \gamma < 3$, at times it may lie outside these bounds. Scale free networks are very robust to errors be optimized for fractal networks. attacks compared to scale-free networks with the same exponent[8]. Thus, it seems that fractality provides better protection when the hubs are removed from the system. This can be attributed to the isolation of the hubs from each other and can provide an explanation on why most biological.

- Scale free networks are very sensitive to targeted attacks, so it is proposed to monitor H, in scale free network traffic flow for detecting attacks. Routing algorithm also select path nodes as per scale free fractal networks for robustness of network.

4. CONCLUSION

Nature adapts Fractal modelling for its creation. Soft computing incorporates fractal properties like fractal dimension, self symmetric, scale invariance in describing and modelling physical phenomenon. Fractal parameters and properties like Space filling, FD, H and self symmetric are used for various analyses. L-system is used for biological modelling. Fractal radio and Fractal Radar concepts are new and requires deep research. Fractal concepts on social networking, Scale free networks and internet are also utilised for robust and fault analysis. Protocols and security system can be designed with fractal parameters. Routing protocol should be designed to select node based on scale free network for better stability of network. Security and network monitoring system can detect an attack by monitoring the network's Hurst parameter. It is proposed to monitor Hurst parameters for scale free network for detection of attack.

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BIOGRAPHIES



Anupam Tiwari was born in Kanpur, U.P. on 9th July 1972, received M Tech (Modeling and Simulation) from Defence Institute of Advanced Technology-(DU), Pune. He had also received BE (Hons) in Electrical Engg and Topper of the course from Assam Engineering College, Guwahati. He received MSc degree (Disaster Mitigation), PG diploma in Thermal Power Plants and a certified "Power Engineer" from CEA. He is a Radiological Safety Officer Level-1 from BARC and Life Member of IETE. He has 19 papers in Journals, and International/National conferences. His field of interest is Modeling and Simulation, Antennas design, Wireless communications and Radar technology.