

ANALYSIS OF VARIOUS URBAN GROWTH MODELS BASED ON CELLULAR AUTOMATA

Shanthi MadhanMohan¹, S.Ajay Kumar², D.Chandrasekhar Reddy³, S.D.V.Prasad⁴, Dr.E.G.Rajan⁵

¹Asst.Prof, CSE, Malla Reddy Engineering College, Hyderabad, AP, India,shanu_shivak@yahoo.com

²Asst.Prof, CSE, Malla Reddy Engineering College, Hyderabad, AP, India,ajay.sgr@gmail.com

³Asst.Prof, CSE, Malla Reddy Engineering College, Hyderabad, AP,India,reddy.daggula@gmail.com

⁴Asst.Prof, CSE, Malla Reddy Engineering College, Hyderabad, AP,India,sdvinmrec@yahoo.co.in

⁵Professor, PRC Pvt.Ltd, Hyderabad, AP, India,rajaneg@yahoo.co.in

Abstract

From many decades there are different modelling techniques have been developed for better understanding and predicting urban expansion, such as Cellular Automata (CA). CA-based models have been commonly used in exploring various urban phenomena, such as urbanization, urban form change, urban growth effect, etc. These models are important and innovative tools that support planning and development of sustainable urban areas and for better understanding the urbanization process. The data requirements for parameterization, calibration and validation of urban models are intense due to the complexity of the models and their objectives. In this paper, we discuss most widely used urban CA models and these are focused on the reproduction of spatial patterns, is the difficulty to model socioeconomic dynamics and decision making processes regarding land use. These models are not concern on the dynamics of multiple categories of urban land that leads to simulation errors. To overcome this drawback we use Agent Based Models (ABM). It is an effective and process based model which is used for modelling the real world applications like urban growth.

Index Terms: Cellular automata, SLEUTH model, FCAUGM model, Agent Based Model, Comparison of Urban models.etc.

1. INTRODUCTION

CA is individual-based models designed to simulate systems in which states, time, and space are discrete. It provides a way of simulating complex systems and self-organizing processes over space and time (Wolfram, 1994). Because of the capabilities of CA, it can able to generate complex patterns through local rules, and for linking rules to their consequences.

CA [1] is a discrete dynamical system that is composed of an array of cells, each of which behaves like a finite-state automaton. Any CA system is composed of four components – cells, states, neighbourhood (Moore, circle ...etc) and transition rules. All interactions are local, with the next state of a cell being a function of the current state of itself and its neighbours. (G.W. Flake's, 2000) CA are models in which contiguous or adjacent cells, such as it may contain a rectangular grid, vary their states- their attributes or characteristics - through the monotonous application of simple rules.

Every entity (in two dimensions represented by a cell) is interacting with the surrounding cells only. Thus, CA has been considered most suitable for processes where the immediate surroundings have an influence on the cell, such as diffusion processes. It has to a large extent demonstrated its capability for modelling complex, self-organization and emergent systems such as urban systems.

CA proposes the advantages of spatiality, dynamics, simplicity and computational efficiency and capability of mimicking real spatial behaviour. It provides an effective spatial-temporal modelling technique for urban dynamics and growth.

Despite the advantages, CA offer in the exploration of the growth of a city a number of errors during the modelling and the simulation may happen. Because of these errors it affect on results of the program. A series of inherent model errors can be identified for CA models in simulation of city growth.

They focus on the following aspects:

- Discrete entities in space and time; (cell size and time for each generation)
- Neighbourhood definitions (types and sizes)
- Model structures and transition rules
- Parameter values and variables (according to the variables and the values the simulation takes into account, some assumptions should be made).

We first discuss the basic concepts of cellular automata and different types of the existing urban growth models such as SLEUTH, FCUAGM and MOLAND with their advantages and disadvantages. Next section we focus on the Agent Based Model (ABM) along with cellular automata for simulating urban growth. Finally, ABM can be used as an effective model for modelling the real world applications like urban growth.

2. VARIOUS EXISTING CA MODELS

2.1. SLEUTH Model

SLEUTH [3], [4], is a eminent simulation model for forecasting urban growth using cellular automata techniques. It uses historical geospatial data for calibration of its parameters. It is composed of a series of growth rules, which form modified Cellular Automata. When running SLEUTH, the rules of the CAs (the growth rules) are calibrated to historical urban spatial data. It used to forecast urban extent under different scenarios. Due to its independent scalability, transportability, and transparency which has become a popular tool in modeling, the increase of urban extent over time and recreating the past or forecasting growth into the future.

The SLEUTH simulation model actually incorporates two different models such as Clarke Urban Growth Model (UGM) and Deltatron Land Use/Land Cover Model (DLM). These models together referred to as *SLEUTH* based on input data such as Slope, Land cover, Exclusion, Urban, Transportation, Hill shade. The SLEUTH software itself can handle three distinct modes of operations: Testing, Calibration, and Prediction [3]. Testing mode is used to create a set of historical data without having the hard constraints on model parameters that calibration does. This mode allows verification of coefficients generated by hand or by calibration processes.

The Calibration process is a more complex procedure used to fine-tune the model parameters by running multiple scenarios with randomized parameter values. This process performs a Monte Carlo simulation of simultaneous growth simulations and emits statistical output to allow hand selection of the most desirable or most accurate input parameters by the investigator. The Prediction process of SLEUTH is also a

Monte Carlo simulation of multiple simulations, but differs from the calibration mode by having the initial state and parameters be identical for each chain of simulations.

2.1.1. Clarke Urban Growth Model (UGM):

The UGM is a cellular automaton which considers urban regions beside with transportation networks and slopes in order to cause predictions of urban growth in an area over time. Clarke *UGM* simulates the combined influences of topography, adjacency, and transportation networks on the patterns of urbanization through time (Clarke et al. 1996). This model uses CA to model the urban expansion based on growth rules in a grid representation of geographic space on a cell-by-cell basis. It is able to control the behaviour of the system by several parameters and by modification of the growth rules. The land cover change deltatron model is tightly coupled with the *UGM*.

Each UGM growth cycle consists of four growth steps. They are Spontaneous Growth, New Spreading Centres, Edge Growth, and Road-Influenced Growth [2]. One cycle of these steps results in the prediction for a year worth of growth. The growth rules considered for each cell during a single cycle are:

a) Spontaneous growth: It reproduces the random low-density growth in isolated cells. If the location has at least one urban neighbour or passes a randomized test of slope suitability, this location becomes a new urban location. Spontaneous growth occurs under the control of the slope resistance factor, which determines the maximum slope value that would allow for development.

b) Diffusive growth: It reproduces the emergence of new spreading centres by creating two neighbouring cells around spontaneous growth areas. The strain coefficient determines the probability of a spontaneous growth cell becoming a new spreading centre.

c) Organic growth: It simulates edge growth in new or existing urban centres. This type of growth occurs under the control of the spread coefficient, which determines the probability that any non-urban cell with at least three urban neighbours will generate an additional urban cell in its neighbourhood.

d) Road-influenced growth: It generates new spreading centres along the roads. To this end, the model randomly selects newly developed cells with a transition probability defined by the breed coefficient, and seeks the existence of a road in the neighbourhood of each cell within a given maximal radius determined by the road gravity coefficient. If a road is

found in the neighbourhood of the selected cell, a temporary urban cell is placed at the point on the road that is closest to the selected cell. Then, this temporary urban cell seeks a permanent location along the road, in a randomly selected direction, and the maximum distance travelled is defined by the diffusion coefficient. The final location of the temporary developed cell is then considered a new urban spreading nucleus, where up to three cells can be developed along the road.

2.1.2. Deltatron Land Use/Land Cover Model (DLM):

The DLM model uses Land use information along with urbanization information from the UGM to model and it shows how land use might transition in the presence of urbanization or other land use change.

The dynamics of the land cover change are distinct through a four-step process: a) Initial Change, b) Cluster Change, c) Propagate Change, d) Age Deltatrons.

a) Initiate Change - when a new urbanization event occurs, that is when a cell is transitioned to be in an urban state in the UGM, then a new potential for land use change is handled. This probability of change is weighted by the average slopes of each class, historical land cover changes forming a transition probability matrix, and the slope of the current cells. Considering these factors define a Random Markov Field for the cell with probabilities for transitioning from the current state of the cell to another.

b) Create Change Cluster - when a new Deltatron is created as a result of a land use change, the probability of the change affecting a larger number of neighboring cells is handled by this rule. The maximum growth of this potential cluster is configurable, and the state to transition to is the same as the initiating land use state.

c) Propagate Change - this rule can be considered to perform the same function as the Edge Growth rules in the UGM model. It defines the probability that adjacent neighbors to a cell will be converted to the same type as a neighbor's land type.

d) Age Deltatrons - Finally, in the Age Deltatrons step all deltatrons are aged to the next time step. The number or cycles a deltatron may live is defined by `min_years_between_transitions`. If they become "older" then this maximum deltatron age, they "die" and can, in principle, be recruited as a new potential deltatron in the next growth cycle

Advantages of SLEUTH Model: Because of variant transition rules, SLEUTH is a self-organizing CA model and, therefore, the coefficients that control growth may vary according to numerous factors. Due to its independent scalability, transportability, and transparency which has become a popular tool in modeling, the increase of urban extent over time and recreating the past or forecasting growth into the future.

Drawbacks of SLEUTH Model: The simulations performed with SLEUTH underestimate the emergence of urban. SLEUTH produces some of the errors, because this model does not consider the dynamics of multiple categories of urban land. The number of patches generated by SLEUTH is much lower and patches are larger and more clustered. This model produces an excess underestimation of new growth and an overestimation of infill growth.

2.2 Fuzzy Cellular Automata Urban Growth Model (FCAUGM):

Fuzzy logic combined to CA provides a proper framework for expressing and mapping the urban growth dynamics. FCAUGM is generally capable of simulating and predicting the complexities of urban growth. This model was shown to be capable of replicating the trends and characteristics of an urban environment. One of the novelties of this model is the design and development of a set of transition rules based on fuzzy set theory rather than probability theory. To replicate realistically the spatial dynamics and processes of urban growth, several transition rules were developed.

These are constructed in two stages: a) by identifying spatially-based factors, b) by creating "driving forces" for urban growth. In Fuzzy Logic, variables consist of partially overlapping qualitative fuzzy sets. Each fuzzy set is described by a linguistic variable familiar to its quality while quantitative (numerical) information is appointed to the proper fuzzy sets by the correspondent membership function. The knowledge base is represented as linguistic "IF...THEN" rules, connecting hypotheses to conclusions through a certainty factor. The most frequently used inference engines are classified into the stages of aggregation, implication and accumulation. Aggregation returns the fulfilment of the hypothesis for every rule individually. Implication combines aggregation's result to the rule's certainty factor (CF) resulting to the degree of fulfilment for each rule's conclusion. Finally accumulation corresponds to compromising different individual conclusions into a final result.

2.2.1 Structure of the FCAUGM:

The model is comprised of three interlinked software modules such as calibration, simulation and prediction. The framework and these modules are designed to be generic, flexible and extensible. Along with these three modules form a prototype Spatial Planning Support System (SPSS)[4].

This urban model differs from earlier work by its use of automated calibration routines including genetic algorithms and simulated annealing and by its construction of transition rules using fuzzy logic. These methods provide much efficient and accurate means of calibrating the model. The inclusion of fuzzy logic allowed for the formulation of more realistic transition rules. It is allowed for a much more generic, extensible and adaptable approach than previous methods. This model is rigorously validated through the application of quantitative and qualitative methods. These methods were selected based on their capability to assess changes in spatial structure over time.

The FCAUGM was tested by simulating the spatial patterns of urban growth over several different development stages using the calibrated values. An expert system based on fuzzy logic [5] (Netweaver software) and EMDS [5] (Ecosystem Management Decision Support) has been implemented to help in the process of making decisions related to locating industrial areas. A GIS platform has been used to spatially analyze the suitability of a municipality to locate an industrial area, and considering the fuzzy logic attributes.

Advantages of FCUAGM: The fuzzy logic gives the evaluation closer to the complex reality of regional planning. Assignment of the weights to the different indicators can be taken into consideration in an environmental impact using fuzzy logic, in order to obtain a significant homogeneity and objectivity.

Disadvantage of FCUAGM: Direct employment of fuzzy logic lies in the way knowledge is captured, i.e. by employing man-made rules. The construction of a manual, expertly guided rule-base is a complex task due to the presence of a high number of inter-dependent variables.

2.3 The MOLAND model:

In MOLAND [12], three types of land use classes are considered instead of two. In fixed land uses, it affects the dynamics of other land use classes but do not change state. In passive land uses, it affects the dynamics of the other classes and change state but they are not influenced by the demands for land external to the model. In active land uses, participate in the dynamics, change state and are influenced by the

demand for land external to the model. In this model, inertia parameters or constraints are not modified from land use class to another. Except for fixed land uses, the rest of land uses can change state freely until the amount of change simulated for each land use and iteration is satisfied.

Advantages of MOLAND Model: MOLAND is the model that comes nearest to real percentages of each type of growth and it has produced growth patterns closer to the real ones than the other models. Indeed, it produced more realistic urban patterns near the road network than the model of White et al. since it considered various types of roads (consequently, various coefficients) in the calculation of the effects of road type on the transition potential, instead of a single type of road and, consequently, a single coefficient.

Drawbacks of MOLAND model: The cell-to-cell correspondence between the real and the map simulated using MOLAND is low. Yet, establishing the correct location of each simulated land use cell is very difficult because of path dependence and stochastic uncertainty.

2.4. Limitations of different Cellular Automata models:

There are some common limitations for all CA-based models, they focused on the reproduction of spatial patterns and this is the obscurity to model socioeconomic dynamics and decision making processes regarding land use. None of these models consider the dynamics of multiple categories of urban land and they may produce simulation errors such as those found in the visual inspection of results, caused by the lack of differentiation between a single-family detached house with a large garden and a group of single-family detached houses. These shortcomings are overcome by Agent Based Models (ABM), which are process based.

3. AGENT BASED MODEL:

The ABM can be used to overcome the limitations of existing models. The proposed model ABM is used for aspatial dynamics and CA is used for spatial dynamics. The major objective of the study is to examine the feasibility and utility of implementing agent-based models with a Geographic information system in order to simulate selected urban growth processes. In this model, system's dynamic behaviour is represented through rules governing the actions of a number of autonomous agents. An agent-based model is a generalization of cellular automata in which agents are able to move around in space, rather than being confined to the cells of a raster.

An agent-based model gives an idea about the collective phenomenon emergence and the individual interactions and processes, which led to the origin of the aggregate phenomenon. The individual level interactions and processes help understand the dynamics that drive and influence an emergent phenomenon. Agent-based modelling is clearly distinguished from other kinds of modelling research by this focus on the concept of agents. An Agent is defined as a computational entity such as a software program or robot that can be viewed as perceiving and acting upon its environment and that is autonomous in that its behaviour at least partially depends on its own experience. An agent can be a system that decides for it what it needs to do in order to satisfy its objectives.

Cellular automata (CA) based models and agent based models (ABM) [8] are flourishing in the present trend. The increasing use of AI approaches has led to a new generation of urban growth models, in which dynamic models based on fine-scale cells and individual behaviours involving agents has begun to enhance the existing interaction and synchronization between different scales over the model and capture the emergent phenomena resulting from the interactions of individual entities.

Agent-based models are useful in conceptualizing land use changes and urban growth. Each agent, in such models, acquires its momentum from factors like the configuration of the land use of its neighbours, the cost of living, cost of transportation (Bid-Rent4), accessibility and other factors determining the quality of life. The spatial environment in the model includes land use attributes (slope, land use, excluded, urban, transportation, hill shade), land price distribution, surrounding environment. A spatial environment includes a virtual real estate market, social network, government policies and casualty problems.

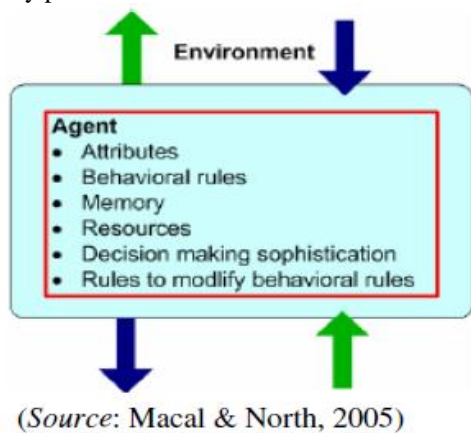


Fig - 1: Agent

Urban land change phenomena include spatial and aspatial dynamics. So we defend that the inclusion of CA for spatial dynamics and ABMs for aspatial dynamics is a better solution for urban modelling. CA has evolved greatly from its initial concepts, many functions have been improved (e.g., action at a distance, calibration and definition of transition rules) to make CA more flexible and efficient approach for urban studies.

However, CA because of its poor spatial representation it has its own limitation. CA lacks the ability to reflect the feedback of system and social economic influence on decision making. Agent based models has the ability to represent the impacts of autonomous, heterogeneous, and decentralized human decision making on the landscape and this can be incorporated along with CA for improving it. Thus, the hybrid model, which is composed of CA and ABM, is a more appropriate method for urban modeling since it possesses the advantages of both CA and ABM (Nara and Torrens, 2005)

3.1 Integration of Cellular Automata Model and Agent Model:

An Agent-based Cellular Automata (ABCA), which combines CA and agent-based models. In the ABCA framework, the object-oriented approach to cells is combined with the transition rules defined by the models as automata. The agent-based models defining the transitional rules are called as agent-automata.

A formal definition of the ABCA [8] ,[9] is deduced from the traditional CA transitional rule. state, s , and neighborhood, N , of the cell at time, t , to define the transition to the state at time, $t+1$. It represents for the discrete time-stepped simulation of the entire region from time t to $t+1$. The agent automata are those, which can be as many in number for the region with varied spatio-temporal characteristics. The variation in space is to denote the sphere of activity or influence of the agent automata and the temporal variability indicates the discreteness of the agent automata and the different start and end time of the agent automata. In this regard, the agent automata are considered as distinct.

CA model for simulating urban growth is configured by considering some of the drivers. With the basis of such a CA model, it can be seen that although the model can simulate for the future scenarios, the model is yet incapable of directly interacting with the drivers dynamically.

For example, in the event that there is a sudden upsurge in population in one of the regions, the present configured model is incapable of addressing such scenarios. The transition rules of the CA model, so defined are incompetent to reflect the

dynamic changes of the drivers by not interacting with them directly. Further the transition rules are system wide and so much of the details are generalized in the overall model.

3.2 Agent-based Models in Simulating Urban growth:

In view of the above-mentioned limitations of the CA based models, agent-based models are used to show that this model can act as a synthetic interface to the dynamic drivers of the system and the simulation framework. The key characteristics of the agent include, autonomous, social ability, responsiveness, and proactive. These agents are diffused into the CA model, which then initiate transitions; respond to the transition and exchange and report accordingly to the properties related with each of the agent-based models. Human decision making for agents is made ease by making use of these properties as well as the simulation is more realistic. Thus agents of an agent-based model to initiate transitions would diffuse into the CA transition to enact such functions. Likewise, agents of those agent-based models to respond or react would act accordingly. Feedback Loops are used along with CA transition for making interactions among the drivers. The agent based transition rules are combined into the CA transition rules and the resulting final transitions based on the feedback loops. The feedback loops are associated with the different agents and according to their behaviours they are modelled. Subsequently, the final transition rule gets the update from these agents before updating the cell state in the subsequent iterations apart from the inherent CA transition rules [13].

An important aspect which is to be addressed involving these feedback loops in a geo-spatial discrete-time model is the capabilities of these different models. These models have to represent the dynamics and respond to them at the respective spatial and temporal scales of the models. Each of the agent-based models representing the drivers is considered as a discrete-time stepped model, while the general CA being another discrete time stepped model both are similar but with time advancement mechanisms. Synchronization is an important aspect that has to be ensured while dealing with these different space and time variant models.

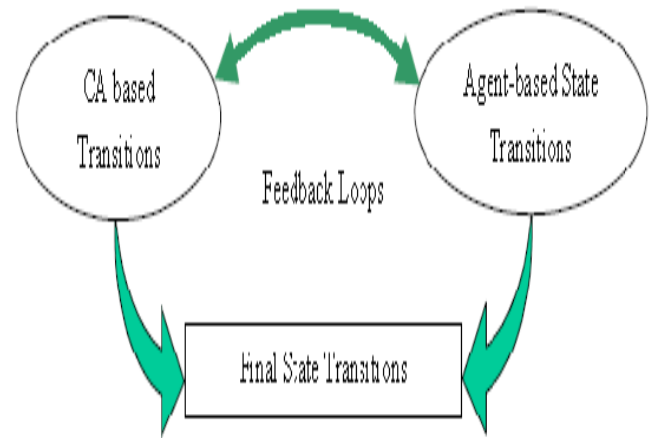


Fig - 3: Feedback Loops

3.3 Scope and Limitations:

The association of agent automata and CA offers more opportunities for geo-spatial modelling and simulation. Agent based model approach can be able to solve the limitation of CA, to respond to drivers and to various externalities dynamically. The ABCA is limited by the consistency of the input data sets and the type of relationships, which are modelled amongst agents. The key conditions for the integration of these agent-based models with CA models are that the spatio-temporal extents of these models/processes are to be predefined. This framework ABCA is more robust to tackle, analyze, test and evaluate the different geo-spatial processes dynamically at discrete space and time [14].

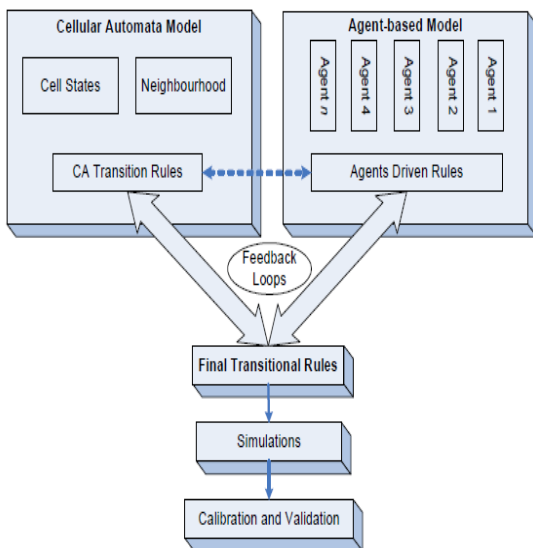


Fig - 2: Combining Cellular Automata and ABM (ABCA)

4. USEFUL TOOLS FOR AGENT-BASED MODELLING:

With the intense research in the realm of agent-based modelling under the distributed artificial intelligence domain, scores of tools are developed for building ABMs, in particular by making use of the programming languages C, Objective C and Java.

a) STARLOGO: Developed by Massachusetts Institute of Technology (MIT) Media Laboratory (<http://www.media.mit.edu/starlogo/>), StarLogo is a programmable modeling environment for exploring the workings of decentralized systems that are self-organizing and self-coordinating. A central notion of the StarLogo is that it consists of three elements – turtles as agents, patches as cells and worldview as observer. With StarLogo, it is possible to model many real-life phenomena, such as bird flocks, traffic jams, ant colonies, and market economies. StarLogo is a specialized version of the Logo programming language.

b) SWARM: Developed by the Santa Fe Institute (<http://www.santafe.edu>), SWARM (<http://www.swarm.org/>) is a software package for multi-agent simulation of complex systems. The basic architecture of Swarm incorporates the collections of concurrently interacting agents. Swarm is essentially a collection of software libraries, written in Objective C, for constructing discrete event simulations of complex systems with heterogeneous elements or agents.

c) REPASt: Stands for REcursive Porous Agent Simulation Toolkit (<http://repaSt.sourceforge.net/>), which is an open source, agent-based simulation toolkit for creating agent-based simulations using Java (1.4 or higher). RePaSt provides a library of classes for creating, running, displaying and collecting data from an agent based simulation. RePaSt has much of the functionalities that are borrowed from the Swarm simulation toolkit and could be termed as "Swarm-like".

d) ASCAPE: Developed by Centre on Social and Economic Dynamics (CSED), Brookings Institution (<http://www.brook.edu/es/dynamics/models/ascape/default.htm>), Ascape (Agent-Landscape) is a research tool to support agent-based modelling and simulation. A high-level framework supports complex model design, while end-user tools make it possible for non-programmers to explore many aspects of model dynamics. It is written entirely in Java, and run on Java-enabled platform.

e) NetLogo: This is a cross-platform multi-agent programmable complexity modelling environment. (<http://ccl.northwestern.edu/netlogo/>). NetLogo comes with a large library of sample models and code examples that help beginning users get started authoring models.

NetLogo was favoured because it was user-friendly and supported extensive documentation for building models.

f) RAISE is object-oriented in design and is implemented in C++. RAISE also features dynamic plugging ability of user-authored rule sets (including easy merging and updating), and development-time plugging ability of reasoning engines.

g) ABLE: The *Agent Building and Learning Environment* (ABLE), (<http://www.alphaworks.ibm.com/tech/able>), is a Java framework, component library, and productivity toolkit for building intelligent agents using machine learning and reasoning. The ABLE framework provides a set of Java interfaces and base classes used to build a library of JavaBeans called AbleBeans.

h) ZEUS: The ZEUS toolkit, which provides a library of software components and tools that facilitate the rapid design, development and deployment of agent systems. The three main functional components of the ZEUS toolkit are: the agent component library, agent building tools and the visualization tools.

i) JADE: Java Agent DEvelopment Framework (JADE) is a software framework to develop agent-based Applications. The goal is to simplify the development while ensuring standard compliance through a comprehensive set of system services and agents.

5. CONCLUSION

Cellular automata (CA) modeling is one of the recent advances in spatial-temporal modelling techniques in the field of urban growth dynamics. It is commonly used in exploring various urban phenomena, such as urbanization, urban form change, urban growth effect, etc. These models provide novel tools that support for better understanding of the urbanization process. In this paper, different types of existing urban models based on cellular automata principles were discussed along with the advantages and disadvantages. The limitations of the existing models are overcome by using the proposed model Agent based Model (ABM). In contrast, ABM possesses more advantageous features for simulating urban development process. ABMs would certainly provide a more realistic representation of complex urban organization, as well as provide us the flexibility to vary urban quantities and population characteristics. Finally, ABM can be used as an effective model for modeling the urban growth dynamics.

REFERENCES

- [1] S.Wolfram, *Cellular Automata and Complexity*, Addison-Wesley Publishing Company, 1994
- [2] Wolfram, S. (2002). *A New Kind of Science*. Wolfram Media, Inc., Champaign, IL
- [3] Simulation of urban growth using a cellular automata-based model in a developing nation's region Changlin YIN , Dingquan YU, Honghui ZHANG, Shengjing YOU, Guanghui CHEN

[4] Applying SLEUTH for Simulating Urban Expansion of Beijing Wang Yi, Bing He

[5] Modeling Urban Growth Dynamics using Cellular Automata and GIS Khalid Al-Ahmadia*, Alison Heppenstallb, Linda Seeb and Jim Hoggb

[6] Modeling Urban Growth using Fuzzy Cellular Automata AGILE International Conference on Geographic Information Science 2008 Lefteris A. Mantelas, Poulicos Prastacos, Thomas Hatzichristos

[7] Lanzhou Urban Growth Prediction Based on Cellular Automata Yaowen Xie*, Aigong Ma, Haoyu Wang

[8] Framework for Integration of Agent-based and Cellular Automata Models for Dynamic Geospatial Simulations H. S. Sudhira, T. V. Ramachandra, Andreas Wytzisk & C. Jeganathan, March 2005, Technical Report: 100

[9] Development of the Integration Framework for CA and Agent-based Models.

[10] Parallelizing a Cellular Automaton Model of Urban Growth Prediction with OpenCL, Jason B. Smith

[11] Developing a Cellular Automaton Model of Urban Growth Incorporating Fuzzy Set Approaches, Yan Liu

[12] A comparative analysis of cellular automata models for simulation of small urban areas in Galicia, NW Spain Andrés M. García, Inés Santé, Marcos Boullón, Rafael Crecente.

[13] Agent-Based Models Of Land-Use and Land-Cover Change Proceedings of an International Workshop, Irvine, California, USA

[14] Agent Based Modeling and Simulation: An Informatics Perspective Stefania Bandini, Sara Manzoni and Giuseppe Vizzari (2009)