DOMAIN OF COMPLEX GROWTH RATE IN COUPLE-STRESS FLUID IN THE PRESENCE OF MAGNETIC FIELD

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Abstract

The thermal instability of a couple-stress fluid acted upon by uniform vertical magnetic field and heated from below is investigated. Following the linearized stability theory and normal mode analysis, the paper through mathematical analysis of the governing equations of couple-stress fluid convection with a uniform vertical magnetic field, for the case of free and perfectly conducting boundaries shows that the complex growth rate σ *of oscillatory perturbations, neutral or unstable for wave number* $a^2 > 0$*, must lie inside a semi-circle*

$$|\sigma|^2 = \left[\frac{Rp_2}{p_1\pi^2\left(8Fp_2\pi^2 + 4p_2 + 1\right)} \right]^2,$$

in the right half of a complex σ*-plane, where R is the Rayleigh number,* p_1 *is the thermal Prandtl number,* p_2 *is the magnetic Prandtl number and F is the couple-stress parameter, which prescribes the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in the couple-stress fluid heated from below in the presence of uniform vertical magnetic field, and it significantly improves the bounds for complex growth of perturbation to Banyal and Khanna (2012).*

Key Words: *Thermal convection; Couple-Stress Fluid; Magnetic field; Complex growth rate; Chandrasekhar number.*

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1. INTRODUCTION

In the subject matter like hydrodynamic stability where experiments has led to the theory all along, the main source of arriving at a mathematical breakthrough is to have a feeling for the right result that may have been suggested from nowhere or through the application of intuitive reasoning based on experience and observation. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics etc. Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics etc. A detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar (1981). The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Sharma et al(1976) has considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics. The fluid has been considered to be Newtonian in all above studies. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stokes (1966) proposed and postulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. According to the theory of Stokes (1966), couple-stresses are found to appear in noticeable magnitude in fluids having very large molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluid, Walicki and Walicka(1999) modeled synovial fluid as couple-stress fluid in human joints. Sharma and Thakur (2000) have studied the thermal convection in couple-stress fluid in porous medium in hydromagnetics. An electrically conducting couple-stress fluid heated from below in porous medium in the presence of uniform horizontal magnetic field has been studied by Sharma and Sharma (2001). Sharma and Sharma (2004) and Kumar and Kumar (2011) have studied the effect of dust particles, magnetic field and rotation on couple-stress fluid heated from below and for the case of stationary convection, found that dust particles have destabilizing effect on the system, where as the rotation is found to have stabilizing effect on the system, however couple-stress and magnetic field are found to have both stabilizing and destabilizing effects under certain conditions.

Banerjee et al (1981) gave a new scheme for combining the governing equations of thermohaline convection which is shown to lead to bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries. However no such result existed for non-Newtonian fluid configurations, in general and for couple-stress fluid configurations, in particular. Banyal (2011) have characterized the non-oscillatory motions in couple-stress fluid.

Keeping in mind the importance of non-Newtonian fluids, the present paper is an attempt to prescribe the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude, in a layer of incompressible couple-stress fluid heated from below in the presence of uniform vertical magnetic field opposite to force field of gravity, when the bounding surfaces of infinite horizontal extension, at the top and bottom of the fluid are free and the region outside the fluid is perfectly conducting. The result is important since the exact solutions of the problem investigated in closed form, are not obtainable.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Considered an infinite, horizontal, electrically conducting, incompressible couple-stress fluid, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0, ρ_0 respectively and at the lower surface z = d are T_d, ρ_d and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The fluid is acted on by a uniform vertical magnetic field $\vec{H}(0,0,H)$ and gravity force $\vec{g}(0,0,-g)$. Let ρ, p, T and $\vec{q}(u,v,w)$ denote respectively the density, pressure, temperature and velocity of the fluid. Then the equation of motion, continuity and heat conduction of couple-stress fluid (Stokes (1966), Chandrasekhar (1981) and Scanlon and Segel (1973)) in hydromagnetics are

$$\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=-\frac{1}{\rho_0}\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)$$
$$+\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}, \qquad (1)$$

$$\nabla.\vec{q}=0, \qquad (2)$$

$$\frac{d\vec{H}}{dt}=\left(\vec{H}.\nabla\right).\vec{q}+\eta\nabla^2\vec{H}, \qquad (3)$$

And

$$\nabla.\vec{H}=0. \qquad (4)$$

The equation of state

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right], \qquad (5)$$

Where the suffix zero refer to the values at the reference level z = 0. Here η stands for electrical resistivity.

Let c_v denote the heat capacity of the fluid at constant volume. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives

$$\rho_0 c_v\left(\frac{\partial}{\partial t}+\vec{q}.\nabla\right)T=\vec{q}\nabla^2 T,$$

Or

$$\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\kappa\nabla^2 T, \qquad (6)$$

The kinematic viscosity ν, couple-stress viscosity μ', thermal diffusivity κ, and coefficient of thermal expansion α are all assumed to be constants.

The basic motionless solution is

$$\vec{q}=(0,0,0), \quad T=T_0-\beta z, \quad \vec{H}=(0,0,H) \text{ and}$$
$$\rho=\rho_0(1+\alpha\beta z). \qquad (7)$$

Assume small perturbations around the basic solution and let $\delta\rho$, δp, θ, $\vec{q}(u,v,w)$, and $\vec{h}=(h_x,h_y,h_z)$ denote respectively the perturbations in density, pressure p, temperature T, couple-stress fluid velocity (0,0,0) and magnetic field respectively. The change in density $\delta\rho$ caused mainly by the perturbation θ in temperature is given by

$$\delta\rho = -\alpha\rho_0\theta . \quad (8)$$

Then the linearized hydromagnetic perturbation equations are of the couple-stress fluid becomes

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}\,\alpha\theta + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} + \left(\nabla\times\vec{h}\right)\times\vec{H}, \quad (9)$$

$$\nabla.\vec{q} = 0, \quad (10)$$

$$\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \quad (11)$$

$$\frac{d\vec{h}}{dt} = \left(\vec{H}.\nabla\right)\vec{q} + \eta\nabla^2\vec{h}, \quad (12)$$

$$\nabla.\vec{h} = 0, \quad (13)$$

Where

$$\kappa = \frac{q}{\rho_0 c_v}.$$

Within the framework of Boussinesq approximation, equations (9) and (12), on using equations (10) and (13), gives

$$\frac{\partial}{\partial t}\nabla^2 w = \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^4 w$$
$$\frac{\mu_e H}{4\pi\rho_0}\nabla^2\left(\frac{\partial h_z}{\partial z}\right) + g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right), \quad (14)$$

$$\left[\frac{\partial}{\partial t} - \eta\nabla^2\right]h_z = H\frac{\partial w}{\partial z}, \quad (15)$$

Together with (11), where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

3. Normal Mode Analysis

Analyzing the disturbances into normal modes, we assume that the Perturbation quantities are of the form

$$[w, \theta, h_z] = [W(z), \Theta(z), K(z)]\ \text{Exp}\left(ik_x x + ik_y y + nt\right),$$
16)

Where k_x, k_y are the wave numbers along the x and y-directions respectively $k = \left(k_x^2 + k_y^2\right)^{\frac{1}{2}}$, is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using (16), equations (14), (15) and (11), on using (10) and (13), in non-dimensional form, become

$$\left(D^2 - a^2\right)\left[\sigma + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]W = -\frac{g\alpha d^2 a^2\Theta}{\nu} + \frac{\mu_e H d}{4\pi\rho_0\nu}D\left(D^2 - a^2\right)K, \quad (17)$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -\left(\frac{Hd}{\eta}\right)DW, \quad (18)$$

$$\left(D^2 - a^2 - p_1\sigma\right)\Theta = -\frac{\beta d^2}{\kappa}W, \quad (19)$$

Where

$$a = kd, \sigma = \frac{nd^2}{\nu}, p_1 = \frac{\nu}{\kappa}, p_2 = \frac{\nu}{\eta}, F = \frac{\mu'}{\rho_0 d^2\nu},$$

$D = \frac{d}{dz}$ and $D_\oplus = dD$ and dropping $(\oplus)$ for convenience. Here $p_1 = \frac{\nu}{\kappa}$, is the thermal prandtl number, $p_2 = \frac{\nu}{\eta}$, is magnetic prandtl number and F is the couple-stress parameter.

Applying the transformations, $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$ and $K = \left(\frac{Hd}{\eta}\right)K_\oplus$ in equations (17), (18) and (19) and dropping $(\oplus)$ for convenience, in non-dimensional form becomes,

$$\left(D^2 - a^2\right)\left[\sigma + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]W = -Ra^2\Theta + QD\left(D^2 - a^2\right)K, \quad (20)$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW, \quad (21)$$

$$\left(D^2 - a^2 - p_1\sigma\right)\Theta = -W, \quad (22)$$

Where $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$, is the Chandrasekhar number.

Since both the boundaries are free and the region outside the fluid is perfectly conducting and are maintained at constant temperature, the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (20), (21) and (22) must be solved are

W = D^2W = 0, $\Theta = 0$ and K = 0 at z = 0 and z = 1. (23)

Equations (20)-(22), along with boundary conditions (23), pose an eigenvalue problem for σ and we wish to characterize σ_i when $\sigma_r \geq 0$.

We prove the following theorem:

Theorem: If R $\rangle$ 0, F $\rangle$ 0, Q $\rangle$ 0 $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, K) of equations (20), (21) and (22) together with boundary conditions (23) is that

$$|\sigma| \langle \left[\frac{Rp_2}{p_1\pi^2 \left(8Fp_2\pi^2 + 4p_2 + 1\right)} \right].$$

Proof: Multiplying equation (20) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\sigma \int_0 W^*\left(D^2 - a^2\right)W dz + F\int_0^1 W^*\left(D^2 - a^2\right)^3 W dz$$
$$- \int_0^1 W^*\left(D^2 - a^2\right)^2 W dz$$
$$= -Ra^2 \int_0^1 W^*\Theta dz + Q\int_0^1 W^* D\left(D^2 - a^2\right) K dz, \qquad (24)$$

Taking complex conjugate on both sides of equation (22), we get

$$\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* = -W^*, \qquad (25)$$

Therefore, using (25), we get

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* dz, \quad (26)$$

Also taking complex conjugate on both sides of equation (21), we get

$$\left[D^2 - a^2 - p_2\sigma^*\right]K^* = -DW^*, \qquad (27)$$

Therefore, using (27), we get

$$\int_0^1 W^* D\left(D^2 - a^2\right) K dz = -\int_0^1 DW^*\left(D^2 - a^2\right) K dz,$$
$$= \int_0^1 K\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma^*\right) K^* dz, \qquad (28)$$

Substituting (26) and (28) in the right hand side of equation (24), we get

$$\sigma \int_0^1 W^*\left(D^2 - a^2\right)W dz + F\int_0^1 W^*\left(D^2 - a^2\right)^3 W dz$$
$$- \int_0^1 W^*\left(D^2 - a^2\right)^2 W dz = Ra^2 \int_0^1 \Theta\left(D^2 - a^2 - p_1\sigma^*\right)\Theta^* dz$$
$$+ Q\int_0^1 K\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma^*\right) K^* dz, \qquad (29)$$

Integrating the terms on both sides of equation (29) for an appropriate number of times by making use of the appropriate boundary conditions (23), along with (21), we get

$$\sigma \int_0^1 \left\{ |DW|^2 + a^2|W|^2 \right\} dz +$$
$$F\int_0^1 \left\{ |D^3W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2 \right\} dz$$
$$+ \int_0^1 \left(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2 \right) dz$$

$$= Ra^2 \int_0^1 \left\{ |D\Theta|^2 + a^2|\Theta|^2 + p_1\sigma^*|\Theta|^2 \right\} dz$$
$$- Q\int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2 \right) dz$$
$$- Qp_2\sigma^* \int_0^1 \left(|DK|^2 + a^2|K|^2 \right) dz, \qquad (30)$$

And equating the real and imaginary parts on both sides of equation (30), and cancelling $\sigma_i (\neq 0)$ throughout from imaginary part, we get

$$\sigma_r \int_0^1 \left\{ |DW|^2 + a^2|W|^2 \right\} dz + F\int_0^1 \left\{ |D^3W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2 \right\} dz$$

$$+\int_0^1\left(\left|D^2W\right|^2+2a^2\left|DW\right|^2+a^4\left|W\right|^2\right)dz$$

$$=Ra^2\int_0^1\left\{\left|D\Theta\right|^2+a^2\left|\Theta\right|^2\right\}dz$$

$$-Q\int_0^1\left(\left|D^2K\right|^2+2a^2\left|DK\right|^2+a^4\left|K\right|^2\right)dz-Qp_2\sigma_r\int_0^1\left(\left|DK\right|^2+a^2\left|K\right|^2\right)dz$$

$$+\sigma_r\left[Ra^2p_1\int_0^1\left|\Theta\right|^2dz-Qp_2\int_0^1\left(\left|DK\right|^2+a^2\left|K\right|^2\right)dz\right], \quad (31)$$

And

$$\int_0^1\left\{\left|DW\right|^2+a^2\left|W\right|^2\right\}dz=-Ra^2p_1\int_0^1\left|\Theta\right|^2dz+Qp_2\int_0^1\left(\left|DK\right|^2+a^2\left|K\right|^2\right)dz$$

(32)

Equation (32) implies that,

$$Ra^2p_1\int_0^1\left|\Theta\right|^2dz-Qp_2\int_0^1\left(\left|DK\right|^2+a^2\left|K\right|^2\right)dz, \quad (33)$$

is negative definite and also,

$$Q\int_0^1\left\{\left|DK\right|^2+a^2\left|K\right|^2\right\}dz\geq\frac{a^2}{p_2}\int_0^1\left|W^2\right|dz, \quad (34)$$

We first note that since W and Z satisfy $W(0)=0=W(1)$ and $K(0)=0=K(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality (1973) $\int_0^1\left|DW\right|^2dz\geq\pi^2\int_0^1\left|W\right|^2dz$,

(35)

And

$$\int_0^1\left|DK\right|^2dz\geq\pi^2\int_0^1\left|K\right|^2dz, \quad (36)$$

Further, for $W(0)=0=W(1)$ and $K(0)=0=K(1)$, Banerjee et al. (1992) have shown that

$$\int_0^1\left|D^2W\right|^2dz\geq\pi^2\int_0^1\left|DW\right|^2dz$$

and

$$\int_0^1\left|D^2K\right|^2dz\geq\pi^2\int_0^1\left|DK\right|^2dz, \quad (37)$$

Further, since $D^2W(0)=0=D^2W(1)$, it follows that

$$\int_0^1\left|D^2W\right|^2dz=\text{Real part of}\left[-\int_0^1DW^*D^3Wdz\right],$$

$$\leq\left|-\int_0^1DW^*D^3Wdz\right|,$$

$$\leq\left|\int_0^1DW^*D^3Wdz\right|,$$

$$\leq\int_0^1\left|DW^*D^3W\right|dz,$$

$$\leq\int_0^1\left|DW^*\right|\left|D^3W\right|dz,$$

$$\leq\int_0^1\left|DW\right|\left|D^3W\right|dz,$$

$$\leq\left[\int_0^1\left|DW\right|^2dz\right]^{\frac{1}{2}}\left[\int_0^1\left|D^3W\right|^2dz\right]^{\frac{1}{2}},$$

(Utilizing Cauchy- Schwartz-inequality),

$$\leq\frac{1}{\pi}\left[\int_0^1\left|D^2W\right|^2dz\right]^{\frac{1}{2}}\left[\int_0^1\left|D^3W\right|^2dz\right]^{\frac{1}{2}},$$

(Utilizing inequality (37)),

So that we have

$$\left[\int_0^1\left|D^2W\right|^2dz\right]^{\frac{1}{2}}\leq\frac{1}{\pi}\left[\int_0^1\left|D^3W\right|^2dz\right]^{\frac{1}{2}},$$

This yield

$$\int_0^1\left|D^3W\right|^2dz\geq\pi^2\int_0^1\left|D^2W\right|^2dz, \quad (38)$$

Using inequality (35) and (37), inequality (38) becomes

$$\int_0^1\left|D^3W\right|^2dz\geq\pi^6\int_0^1\left|W\right|^2dz, \quad (39)$$

Further, multiplying equation (22) and its complex conjugate (25), and integrating by parts each term on right hand side of the resulting equation for an appropriate number of times and making use of boundary conditions on Θ namely $\Theta(0)=0=(1)$ along with (22), we get

$$\int_0^1\left|\left(D^2-a^2\right)\Theta\right|^2dz+2p_1\sigma_r\int_0^1\left(\left|D\Theta\right|^2+a^2\left|\Theta\right|^2\right)dz,$$

$$+ p_1^2|\sigma|^2\int_0^1|\Theta|^2dz = \int_0^1|W|^2dz, \quad (40)$$

since $\sigma_r \geq 0, \sigma_i \neq 0$ therefore the equation (40) gives,

$$\int_0^1\left|\left(D^2-a^2\right)\Theta\right|^2dz \langle \int_0^1|W|^2dz, \quad (41)$$

And

$$\int_0^1|\Theta|^2dz \langle \frac{1}{p_1^2|\sigma|^2}\int_0^1|W|^2dz, \quad (42)$$

It is easily seen upon using the boundary conditions (23) that

$$\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz =$$

Real part of $\left\{-\int_0^1\Theta^*\left(D^2-a^2\right)\Theta dz\right\}$

$$\leq\left|\int_0^1\Theta^*\left(D^2-a^2\right)\Theta dz\right|,$$

$$\leq\int_0^1\left|\Theta^*\left(D^2-a^2\right)\Theta\right|dz,$$

$$\leq\int_0^1\left|\Theta^*\right|\left|\left(D^2-a^2\right)\Theta\right|dz,$$

$$=\int_0^1|\Theta|\left|\left(D^2-a^2\right)\Theta\right|dz,$$

$$\leq\left\{\int_0^1|\Theta|^2dz\right\}^{\frac{1}{2}}\left\{\int_0^1\left|\left(D^2-a^2\right)\Theta\right|^2dz\right\}^{\frac{1}{2}}, \quad (43)$$

(Utilizing Cauchy-Schwartz-inequality)

Upon utilizing the inequality (41) and (42), inequality (43) gives

$$\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz \leq \frac{1}{p_1|\sigma|}\int_0^1|W|^2dz, \quad (44)$$

Now R 〉 0, $Q\rangle 0$ and $\sigma_r \geq 0$, thus upon utilizing (33) and the inequalities (34)-((39) and (44), the equation (31) gives,

$$I_1+\left[F\left(\pi^2+a^2\right)^3+\left(\pi^2+a^2\right)^2+\frac{a^2\pi^2}{p_2}-\frac{Ra^2}{p_1|\sigma|}\right]\int_0^1|W|^2dz\langle 0 \quad (45)$$

Where

$$I_1=\sigma_r\int_0^1\left\{|DW|^2+a^2|W|^2\right\}dz++Qa^2\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

$$+Qp_2\sigma_r\int_0^1\left(|DK|^2+a^2|K|^2\right)dz$$

is positive definite.

And therefore , we must have

$$\frac{R}{p_1|\sigma|}-\frac{\pi^2}{p_2}\rangle\frac{\left(\pi^2+a^2\right)^2}{a^2}\left\{F\left(\pi^2+a^2\right)+1\right\}, \quad (46)$$

and thus we necessarily have

$$\frac{R}{p_1|\sigma|}-\frac{\pi^2}{p_2}\rangle 4\pi^2\left(2F\pi^2+1\right)$$

Since the minimum value of $\frac{\left(\pi^2+a^2\right)^2}{a^2}$ is $4\pi^2$ for $a^2=\pi^2\rangle 0$.

Hence, if $\sigma_r \geq 0$ and $\sigma_i \neq 0$, then

$$|\sigma|\langle\left[\frac{Rp_2}{p_1\pi^2\left(8Fp_2\pi^2+4p_2+1\right)}\right] \quad (47)$$

And this completes the proof of the theorem.

CONCLUSIONS

The inequality (47) for $\sigma_r \geq 0$ and $\sigma_i \neq 0$, can be written as $\sigma_r^2+\sigma_i^2\langle\left[\frac{Rp_2}{p_1\pi^2\left(8Fp_2\pi^2+4p_2+1\right)}\right]^2$

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of couple-stress fluid of infinite horizontal extension heated form below, having top and bottom bounding surfaces free and the region outside the fluid is perfectly conducting, in the presence of uniform vertical magnetic field parallel to the force field of gravity, the complex growth rate of an arbitrary oscillatory motions of growing amplitude, lies inside a semi-circle in the right half of the $\sigma_r\,\sigma_i$ - plane whose centre is at the origin and radius is equal to $\left[\frac{Rp_2}{p_1\pi^2\left(8Fp_2\pi^2+4p_2+1\right)}\right]$, where R is the Rayleigh number, p_1 is the thermal Prandtl number, p_2 is the magnetic Prandtl number and F is the couple-stress parameter, and it significantly improves the bounds for complex growth of perturbation rate to Banyal and Khanna (2012).

REFERENCES

1. Banerjee, M.B., Gupta, J.R. and Prakash, J. (1992). On thermohaline convection of Veronis type, J. Math. Anal. Appl., Vol.179, No. 2 pp. 327-334.
2. Banerjee, M.B., Katoch, D.C., Dube,G.S. and Banerjee, K. (1981). Bounds for growth rate of perturbation in thermohaline convection. Proc. R. Soc. A378, 301-04
3. Banyal, A.S., (2011) A characterization of non-oscillatory motions in couple-stress fluid in the presence of suspended particles (2011), J.Comp. and Math. Scis.(JCMS), Vol.2(3), pp. 537-545.
4. Banyal, A.S., and Khanna, M., Upper Limits to the Complex Growth Rate in Couple-Stress Fluid in the Presence of Magnetic Field, J. Comp. & Math. Sci., Vol. 3(2), pp.237-247, (2012)
5. Chandrasekhar, S. (1981). Hydrodynamic and Hydromagnetic Stability, Dover Publication, New York
6. Kumar, V. and Kumar, S. (2011). On a couple-stress fluid heated from below in hydromagnetics, Appl. Appl. Math., Vol. 05(10),pp. 1529-1542
7. Scanlon, J.W. and Segel, L.A. (1973). Some effects of suspended particles on the onset of Bénard convection, Phys. Fluids. Vol. 16, pp. 1573-78
8. Schultz, M.H. (1973). Spline Analysis, Prentice Hall, Englewood Cliffs, New Jersy.
9. Sharma, R.C., Prakash, K. and Dube, S.N. (1976).Effect of suspended particles on the onset of Bénard convection in hydromagnetics, J. Math. Anal. Appl., USA, Vol. 60 pp. 227-35.
10. Sharma, R.C. and Thakur, K. D. (2000). Couple stress-fluids heated from below in hydromagnetics, Czech. J. Phys., Vol. 50, pp. 753-58
11. Sharma, R.C. and Sharma S. (2001). On couple-stress fluid heated from below in porous medium, Indian J. Phys, Vol. 75B, pp.59-61.
12. Sharma, R.C. and Sharma, M. (2004). Effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field, Indian J. pure. Appl. Math., Vol. 35(8), pp. 973-989.
13. Stokes, V.K.(1966). Couple-stress in fluids, Phys. Fluids, Vol. 9, pp.1709-15.
14. Walicki, E. and Walicka, A. (1999). Inertial effect in the squeeze film of couple-stress fluids in biological bearings, Int. J. Appl. Mech. Engg., Vol. 4, pp. 363-73

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