
A Study of Probabilistic TopHasteningReply for Random Ordinary Loads

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Abstract

A few representative samples of time history of pedestrian loads are simulated using uniform design first, and then the corresponding peak acceleration response spectra are obtained by dynamic analysis on beam structures with different spans and damping ratios. The spectra which have a certain percentile are obtained by reliability analysis based on response surface method. Then the general formulae of peak acceleration response spectra, which can be used to calculate structural peak accelerations directly, are deduced from parametric analysis of damping ratio and span. Monte Carlo simulation is conducted to validate the precision of this method. The case study shows that compare to the results calculated by the proposed method, the formulae in two widely-used codes such as BS 5400-2:2006, overestimate the peak acceleration of structure with high frequency remarkably and it should be cautious when using them to obtain structural responses.

Key Terms—Random pedestrian loads, Uniform design, Response spectrum, Peak acceleration

1.INTRODUCTION

With the development of building materials, structural form, construction technology and aesthetic standards, there is a trend that ISSN 2319 – 6009 www.ijscer.com Vol. 1, No. 1, November 2012 © 2012 IJSCER. All Rights Reserved Int. J. Struct. & Civil Engg. Res. 2012 Research Paper structures such as footbridges, gymnasium, stadium and airport passage, are designed to be lighter, slender and more flexible. Under pedestrian loads, these structures with lightmass and small damping usually hardly undergo significant structural damage, but excessive vibrations that affect structural serviceability may occur due to inappropriate design. For example, in 1993, Fujino et al. reported that a cable-stayed pedestrian bridge in Japan presented excessive vibration in the congested condition. In 2000, during the opening day of Millennium Bridge in London, its vibration due to pedestrian was so excessive that the bridge needed to be closed on the opening day (Dallard, 2001; and Strogatz et al., 2005). After this incident, the issue of vibration serviceability

of similar structures caused by pedestrian loads has increasingly aroused many scholars' and researchers' attention (Song, 2003 and 2005; Zivanovic, 2005, 2007a and b; Huang et al., 2007; Piccardo et al., 2008; and Fa, 2008). In these literatures, Zivanovic et al. (2005) summarized a comprehensive literature review on vibration serviceability of footbridges under human-induced excitations, including many aspects such as models of human-induced walking force, physical characteristics of footbridges, calculation methods of structural response, criterion of human comfort, etc

In order to estimate vibration serviceability of these structures, pedestrian loads should be obtained to calculate structural vibration response at first, and then according to the criterion of human comfort, evaluation of human comfort in terms of response indexes such as peak acceleration, root mean square (RMS) acceleration, etc., can be conducted. Generally, there are two types of model of vertical pedestrian load: deterministic model and probabilistic model

(Zivanovic, 2005). The deterministic force model, e.g., sinusoidal model, is simple and easy to be applied in subsequent dynamic analysis. As a result, deterministic sinusoidal models of vertical walking load are adopted in some codes and research. Based on a few deterministic time histories of vertical walking loads, Song (2003 and 2005) proposed a convenient method named response spectrum method, to calculate structural peak acceleration response. Silva et al. (2003 and 2007) used a deterministic force model to study vibrations of composite floor and footbridges caused by rhythmic human activities. Figueiredo et al. (2008) compared the structural acceleration responses under four different deterministic models of pedestrian vertical force. The results suggest that the change of location of pedestrian loads should be considered in the calculation of acceleration response

In reality, however, the pedestrian load is a more complex narrow band random process rather than deterministic forces (Zivanovic et al., 2007b and 2011). Therefore, some probabilistic models of vertical walking loads also have been studied to obtain a more accurate structural response that can reflect the randomness property of loads and responses. There are also mainly two types of model: time- and frequency-domain models. For time-domain, Zivanovic et al. (2007a) took into account many variables in pedestrian loads such as walking frequency, step length, Dynamic Load Factor (DLF) and phase of each harmonic component, and subsequently used Monte Carlo Simulation (MCS) to generate 2000 samples of pedestrian loads. They calculated the cumulative probability of RMS acceleration of footbridge by statistical analysis of the 2000 time histories of structural responses corresponding to the 2000 samples of pedestrian loads and then obtained the RMS acceleration with a certain percentile. Similar to the concept in Song (2005), Zivanovic et al. (2007b and 2011) and Wan et al. (2009) provided a few probabilistic design spectra for single walking scenario. In terms of frequency-domain analysis, Eriksson (1994) considered pedestrian loads as a stationary random process and obtained its auto-

spectral density. Li et al. (2010) and Fan et al. (2010) studied the auto-spectral density of vertical walking loads and obtained the RMS acceleration responses.

From the above discussion, it is clear that the walking force is a narrow band random process and the deterministic force model may not reflect its randomness satisfactorily. Moreover, it may overestimate structural acceleration response significantly (Pimentel, 1997 and 2001) Therefore, calculating the structural vibration response by probabilistic force model is more reasonable. Obviously, using MCS can solve this issue very straightforwardly and simply. The conventional probabilistic calculation method using MCS can directly consider all stochastic variables of pedestrian load and be conducted directly, as in Zivanovic et al. (2007a). However, its efficiency is rather low as it generally needs covering a wide range of experimental datum (sample space) and conducting lots of experiments (samples). For example, a reliability analysis with failure probability P_f and relative error of simulation ϵ , the required sampling number of MCS N

Owing to the failure probability P_f of a real structure is often in the range of $10^{-3} \sim 10^{-4}$, it can be deduced from the above Equation (1) that MCS will lead to plenty of calculation effort. Although the increasing advances of computing power and speed, single calculation of issues such as dynamic analysis, finite element model, fluid dynamics model, etc., can make minutes to hours, if not longer. So analyses of these computer-based issues that require a large number of repeated calculations to obtain a reliable result could be difficult within a limited timeframe. Therefore, it can be seen that MCS is time-consuming and inefficient for these issues that the subsequent calculation of each sample requires much time, e.g., the dynamic analysis. The method of frequency-domain analysis has less calculation and is comparatively more efficient with respect to that of time-domain. However, the methods in literature limit to reflecting the stationary vibration response of structure rather than the real transient vibration

response, which is excited by human walking loads in a relative limited time. In addition, the randomness of the excitement location, i.e., the location of foot, is not considered in this method. Consequently, the structure vibration responses based on the method may be different from the one of real responses

In order to solve the above problem efficiently, the uniform experiment design (UD) method is introduced in this paper. UD can provide some representative samples, which can reduce the required number of samples significantly and cut down calculation work remarkably. In this paper, some representative samples of vertical walking loads were defined by UD, and then the peak acceleration responses are obtained by dynamic analysis in time-domain based on these samples of load. The peak acceleration response spectra with certain percentile were obtained by reliability analysis based on Response Surface Methodology (RSM). Some parametric analyses of damping ratio and span were conducted to provide general forms of response spectrum. The MCS was made to validate the precision of this method. Finally, a case study was presented to show how to use the spectra to calculate the related peak acceleration of structure induced by vertical pedestrian loads, together with a comparative analysis about methods in some related currently-used codes of practice to demonstrate that these methods may overestimate the structural response remarkably

UD And RSM

As mentioned in forgoing section, even though the computing power of computer has been improving remarkably, some engineering issues still require a large amount of time to obtain good accuracy. This deficiency may become an obstacle to deal with some issues such as dynamic analysis, finite element analysis and reliability analysis. In order to minimize the computational expense of running these computer analyses for reliability analysis that usually uses traditional MCS, statistical approximation techniques, e.g., experiment design and RSM are becoming widely

used in engineering (Simpson, 2001a). An experiment design systematically selects a sequence of experiments to be performed, which is essential for effective experimentation (Simpson, 2001b) According to these representative experiments, the RSM can form an approximation of the relationship between response/output and a number of input parameters that is accurate enough to replace the original model. Then we can make use of this approximation to make subsequent analysis so that the computationally expensive simulation or calculation is no longer required, which can facilitate analysis and enhance the efficiency remarkably. A more extensive introduction about RSM is given by Box (2007).

UD method is one of the above experiment design methods. This method was proposed by Fan and Wang, which scatters experiment points uniformly in the range of experiment parameters, i.e., design space, and selects a sequence of representative experiment points to organize experiments (Fang, 1994 and 2001). Essentially, it is a type of fractional factorial design with an extra property of uniformity. Similar to orthogonal experimental design (OD), it carries out experiment runs according to a series of specifically designed tables, i.e., UD table. Compare to OD, UD has higher efficiency as it requires fewer sample points and obtain better coverage of design space. UD table is important for select some representative sample set and it is usually expressed as $UN(q_s)$, where U stands for uniform design, N is the number of experiment runs to conduct, q is the level number of each parameter and s is the number of total parameters that the table can contain at most. It is obvious that the remarkable character of UD is that its required number of experiment usually equals the number of parameter levers. This means that the number of experiment organized by UD is much less than that organized by OD or full factorial design. Take an experiment with s parameters and q levels

for each parameter, for instance, UD only needs q experiment runs, while OD needs q^2 runs and full factorial design needs q^s runs. With respect to OD

and full factorial design, UD can improve experiment efficiency significantly, especially for experiment involving more than 3 parameters or levels. Some successful applications are presented in (Fang, 1994, 2001, 2003 and 2008; Liang et al., 2001; Song et al., 2010 and 2012). Fang (1994) also provides many UD tables, which can be used directly for applications.

For an experiment with m variables, X_1, X_2, \dots, X_m , the procedures of UD are listed as follows: Defining the range $[X_{imin}, X_{imax}]$ ($i = 1, 2, \dots, m$) of each variable, where X_{imin} and X_{imax} are minimum and maximum value of the i th parameter, respectively.

where $j = 1, 2, \dots, n$ is the level number; X_{ij} is the j th level of the i th parameter. Levels also can be divided unequally, and the corresponding process can refer to Fang (1994). 3. Choosing a proper UD table to design the experiment The selection of UD table is determined by the numbers of parameters and of levels. We can use appropriate UD table $U_n(nm)$ in Fang (1994) or we can construct UD table according to the methodology in Fang (1994 and 2003). After finishing all experiments, the corresponding response surface can be formed based on the input variable sets and the output.

Simpson et al. (2001) compared four design experiments by examples and the results show that the good design space coverage of UD tends to provide more accurate approximation globally even with a low sample size. Song et al. (2010, 2012) conducted a reliability analysis of compartment fire by adopting UD. The results of 24 sets of experiments run designed by UD present good agreement with the ones of 5,000 MCSs, which can show the efficiency of UD.

SIMULATION OF HUMAN INDUCED WALKING FORCE BASED ON UD

In time-domain, vertical pedestrian walking loads are usually expressed as (Ebrahimpour et al., 1996; Zivanovic et al., 2005 and 2011) where, W is pedestrian weight; DLF_i is dynamic load factor of i th harmonic component; f_s is walking step frequency; θ_i is phase of i th harmonic component. According to Kerr's study (2001), the DLFs of higher harmonic are small and by the fifth harmonic the DLFs are about zero. This paper therefore considers only the first five harmonic components and the dynamic component of vertical walking loads could be expressed as The duration and location of the above load model are also related to step length L_s , so the pedestrian walking loads reconstructed by

the Equation (4) contain 13 parameters totally, i.e., pedestrian weight W , walking step frequency f_s , step length L_s , DLFs and phase θ_i of the first five harmonic component. According to Zivanovic et al. (2007a, 2011) and Kerr et al. (2001), these parameters and their probability distribution are shown in Table 1, which are represented as $X_1 - X_{13}$ hereinafter for ease of referencing.

As the foregoing discussion, it is very straightforward to use MCS method to simulate many samples of vertical walking forces based on Equation (4) and distribution of each parameter in Table 1, as in Zivanovic et al. (2007a) and Fa et al. (2008). However, this method will cause a lot of calculation works for the subsequent dynamic response analysis. Therefore, UD method is introduced here to reduce the required number of samples and solve the issue efficiently.

The steps of UD for this issue are listed as follows. Defining the range $[X_{imin}, X_{imax}]$ ($i = 1, 2, \dots, m$) of each variable. In this paper, the range of each uniform parameter, i.e., the phase, is $[0, 2\pi]$; the range of each normal parameter is

Table 2: Levels of All Parameters

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}
1	1.31	0.50	394	0.52	0	0	0	0	0.00	0.00	0.00	0.00	0.00
2	1.35	0.51	411	0.553	0	0	0	0	0.11	0.11	0.11	0.11	0.11
3	1.39	0.53	428	0.586	0	0	0	0	0.22	0.22	0.22	0.22	0.22
4	1.43	0.54	445	0.619	0	0.048	0.048	0	0.33	0.33	0.33	0.33	0.33
5	1.47	0.56	462	0.652	0.069	0.131	0.131	0	0.43	0.43	0.43	0.43	0.43
6	1.50	0.57	479	0.686	0.158	0.214	0.214	0.017	0.54	0.54	0.54	0.54	0.54
7	1.54	0.59	496	0.719	0.246	0.297	0.297	0.121	0.65	0.65	0.65	0.65	0.65
8	1.58	0.60	513	0.752	0.335	0.379	0.379	0.224	0.76	0.76	0.76	0.76	0.76
9	1.62	0.62	530	0.785	0.424	0.462	0.462	0.328	0.87	0.87	0.87	0.87	0.87
10	1.66	0.63	547	0.818	0.512	0.545	0.545	0.431	0.98	0.98	0.98	0.98	0.98
11	1.70	0.64	564	0.851	0.601	0.628	0.628	0.534	1.08	1.08	1.08	1.08	1.08
12	1.74	0.66	581	0.884	0.69	0.71	0.71	0.638	1.19	1.19	1.19	1.19	1.19
13	1.77	0.67	598	0.917	0.778	0.793	0.793	0.741	1.30	1.30	1.30	1.30	1.30
14	1.81	0.69	615	0.95	0.867	0.876	0.876	0.845	1.41	1.41	1.41	1.41	1.41
15	1.85	0.70	632	0.983	0.956	0.959	0.959	0.948	1.52	1.52	1.52	1.52	1.52
16	1.89	0.72	649	1.017	1.044	1.041	1.041	1.052	1.63	1.63	1.63	1.63	1.63

Once the vertical loads of human walking have been defined, the corresponding structural response, which can be used to evaluate the structural serviceability, can be calculated by structural dynamic analysis. Since the boundary conditions of structures hardly have substantial influence on the structural responses for this type of loads Song (2004) and Song and Jin (2005), only simply supported beam is analyzed here. The equation of motion of the structure under the excitation of single pedestrian is usually expressed their modal form as (Zivanovic et al., 2007, 2007a and 2011; and Wan, 2009),

It should be noted that the location of loads defined by Equations (4) and (7) not only varies with time, but also the loads just excite on some certain points, instead of a continuing force on the beam like a vehicle force model in Equation (7) as shown in Figure 2. Therefore, the force model in Equation (7) may not reflect the actual human walking excitation accurately. In

this paper, a force model that just excites on the locations of foot is proposed as in the following Equation (8), which can reflect the mechanism of walking excitation more accurately. For the beam-type structure shown in Figure 2, the structural response can be calculated according to the following equation of motion

CONCLUSION

UD method is introduced to avoid using Monte Carlo method to generate lots of random samples and to improve the efficiency of solving this type of issue. Then acceleration response spectra with 95% percentile are obtained by reliability analysis based on response surface constructed by only a few representative samples of pedestrian loads defined by UD and the corresponding structural responses calculated by dynamic analysis. The Monte Carlo simulation validates the method has

enough precision. It can be seen that this calculation system based on UD reduces the calculative work significantly and provides a method to simplify the probabilistic dynamic response analysis of structure under random incitement. Once the structure parameters such as f_{st} and M_1 are given, the structural peak acceleration response could be calculated in a rather simple way by using the general equations of peak acceleration response spectra in the paper rather than by time-consuming dynamic calculation, which is of practical application. The case study shows that compare to the results calculated by the proposed method, the formulas in BS 5400-2:2006 and Canadian Highway Bridge Design Code provide a considerable overestimate of the peak acceleration of structure with frequency more than 2.5 Hz and using them to obtain responses of these structures should be cautious.

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